

ADVANCES IN MODELLING TRAFFIC GENERATION

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1. INTRODUCTION

Modelling the generation of traffic, whether we consider the number of trips or tours made by individuals or households, or the flow of goods requiring transport, remains a challenging problem for transport planners. This paper presents some new findings that should help to reduce the difficulty of the problem.

A central issue in modelling generation and the main concern of this paper is to represent the impact of accessibility on trip rates (or freight flows). The correlation between accessibility and traffic volumes has been established at the level of socio-economic variation for many years (car owners travel more than non-car-owners) and at the level of geographical variation in recent years. The latter connection is often weak in urban and regional studies, although there is no doubt that such a connection exists, but accessibility can be quite strong in influencing travel rates in specific corridors, particularly when tourism is involved. When choice models are developed for other aspects of travellers' choices, accessibility can be measured quite simply by a 'logsum' taken from those other models, giving the average utility of travelling. The logsum can incorporate both socio-economic and geographical variation.

In the following section of the paper, we review some of the recent literature in this area, following on from the more extensive review of the earlier literature in an earlier paper (Daly, 1997). Conventional approaches are discussed, followed by work which attempts to improve on those approaches by interpreting the data more correctly and/or by modelling generation for different purposes simultaneously. An important paper by Larsen (2003) reviews model forms and the appropriate units to model, i.e. trips, tours (which he finds superior to trips) or 'visits' which is his preferred unit.

In our earlier work we argued for the representation of the demand for travel either as an explicit choice among exact numbers of trips or tours (0, 1, 2 etc.) or by a log-linear model. The latter, often termed an exponential model because the expected trip rate can be represented as an exponential function, is convenient in application but could not be linked in our earlier work directly to utility theory and so could not be completely justified in a model system intended to be based rigorously on that theory. In the third section of this paper, however, the utility theory underlying the exponential model is established so that the model can be used as part of a complete structure of utility-maximising models.

However, it is also clear that the exponential model form does not fit observed data well.

A further issue in modelling traffic generation is to establish consumer surplus. If there is an improvement in accessibility (measured by a logsum), then current travellers will get the complete benefit of that change – the logsum can be used as a measure of the benefit they will obtain. But newly generated traffic will also obtain benefit, and, although the ‘rule of a half’ can be applied to the logsum improvement to obtain an approximation, it is better and simpler to use an exact calculation of the benefit obtained. In Section 4 a very simple but exact surplus measure for the exponential model is derived and it is shown that this has the properties that would be expected.

In the final main section, attention is devoted to the issue of estimation of these models. The difficulty of using stated choice data is discussed and methods are suggested by which these problems can be overcome. Issues arising in the use of revealed preference data are also discussed and recommendations are made for future practice.

2. LITERATURE AND CURRENT PRACTICE

Conventional approaches to traffic generation treat the data as resulting from a continuous process and deal with each travel purpose separately. An improvement to this approach may be sought by noting that the numbers of trips people make are whole numbers, not negative: 0, 1, 2 etc.. We can make further progress by observing that in many cases an outbound trip is directly followed by a return trip, so that modelling tours or visits for specific purposes is suggested. To make still further improvements we can consider that there are linkages between purposes, either because the fact that someone is working reduces the time they have for shopping or recreation, or that travel for one purpose opens the possibility of ‘trip chaining’ to other destinations for the same purpose or for other purposes.

2.1 Conventional approaches

In our previous work on this subject (Daly, 1997) we reviewed the then current methods: category analysis and various forms of regression, basing much of our discussion on the second edition of the standard transportation modelling textbook, Ortúzar and Willumsen (1994). Since then, a third edition of that work has appeared, which we can reasonably use to indicate the development in the area in the last decade. For a review of the earlier literature, please see our previous work.

The two new features in Ortúzar and Willumsen (2001, reprinted 2004) are an increased emphasis on the possibility of conducting category analysis at person

level, rather than at household level, and a review of our own work. However, no clear recommendations are made as to which methods should be used in which circumstances. The difficulties of the current standard methods outlined in our previous work remain.

In the UK Government's advice to planners and analysts on appropriate modelling procedures (WebTAG, 2006) the modelling of traffic generation is split into two components. First, growth in traffic due to exogenous forces (population, employment, income, etc.) should be forecast, then changes in the amount of traffic resulting from changes in accessibility should be forecast as changes relative to the central forecasts. No advice is given in WebTAG as to how the central forecasts should be made, but the TEMPRO (2006) forecasts are likely candidates. These forecasts are understood to be based on conventional regression methods.

More extensive advice is given in WebTAG (2006) on changes in traffic levels as a consequence of accessibility changes, suggesting (Document 3.10.3, Appendix 2.1.10) that an exponential form is most suitable in conjunction with other choice models of logit form which would generate accessibility measures of the logsum type. This recommendation essentially follows that of our previous work, which indicated the exponential form as the best when an explicit choice model was not used and that the exponential form and explicit choice model with repeated 'geometric' probabilities were close approximations to each other.

2.2 Interpretation of the data

A key issue in the formulation of generation models is whether the dependent variable of the model is interpreted as

- a whole number with some probability or
- a continuous number representing the expectation of the number of trips generated.

The former approach would generally give a more correct interpretation of observed data, while the latter is often more convenient for forecasting. Ideal would be to combine the advantages of both interpretations.

Because of the policy importance of measuring the impact of accessibility on total numbers of trips and the difficulty of estimating that impact, it is essential in the model estimation process to formulate the model optimally for estimation. Conventional approaches (e.g. those outlined in Ortúzar and Willumsen, 1994) have generally not done this and in consequence estimations of the accessibility impact have not been as precise as they might have been. Our previous work therefore focussed on a disaggregate formulation of the model, modelling the probability of choosing among the alternatives 0, 1, 2 .. using repeated binary logit choices as explained in the following section. A key feature of this approach is to note that the choice between 0 and one-or-more tours in a day is of a different type than the subsequent choices, since this choice is whether or not to

participate in an activity in the day, whereas the subsequent choices of 1, 2, 3 etc. tours are about how to organise that participation.

Larsen (2003) discusses possible formulations of a model to address the observations of the numbers of trips, tours or visits made in a fixed period (e.g. a day). After an analysis of Norwegian data he selects visits (what other researchers have termed sojourns) as the unit of study. Larsen also assigns a special role to the choice of zero visits.

Larsen discusses a binomial approach (Bernoulli trials), a structured logit model and a geometric model, before deciding to test Poisson and Logit-Poisson approaches, the latter using a logit model for the choice of whether to make 0 or one-or-more visits, then a Poisson model for the remaining choices. The Logit-Poisson model generally performed better in the Norwegian data he tested, but was not significantly better in all the tests.

2.3 Consideration of multiple purposes

Larsen (2003) discusses the interactions of the numbers of visits for different purposes. He speculates that there may be positive or negative correlations and analyses these by looking at the residuals from the single-purpose models. Overall, negative correlation predominates. He gives some interesting speculations as to how this approach might be turned into a complete model, but does not derive a working model with linked purposes.

More detailed modelling of the generation of travel for multiple purposes, often including some treatment of trip chaining, is given in the growing literature on activity analysis. A review of the relevant aspects of the literature to that date was presented in Daly (1997); a further of developments in this literature would be outside the scope of the present paper, which is concerned with improvements to models for a single purpose.

3. UTILITY BASIS FOR THE EXPONENTIAL MODEL

Although there are plausible reasons to suppose that alternative theories should be investigated, at present the only consistent basis for the specification of travel demand model systems is given by utility theory. Particularly when a linked system is required, including travel generation along with models of other aspects of travel choice, themselves generally based on the concept of utility maximisation, it is natural to require that generation models should also have that basis.

It is now standard practice that linked model systems, based on logit choice models, should be set up on this utility-maximising basis and using logit-type models (WebTAG, 2006). The key linkage between model components in such

systems is the 'logsum' variable, which gives a composite measure of the utility of travel by any member of a choice set and which can be regarded as a measure of accessibility.

Almost all travel demand models have the basic structure of a generative component and a distributive component (van Vuren and Daly, 1996), giving the demand T_j for alternative j as

$$T_j = \sum_s D_s \cdot r_{js}$$

where D_s represents the *number* of trips, tours, shipments etc. made by people (or firms) in segment s ; s would be defined over geographical zones and socio-economic (or industrial) segments such as car availability, income etc.;

r_{js} gives the choice probability for alternative j for decision makers in segment s .

The key distinction between the two components of the system, p and D , are that the probabilities r are always calculated over explicit alternatives. The r 's sum to 1. In contrast, the generative model which yields w has no explicit alternative to travel and no explicit ceiling on its value. This is part of the difficulty of modelling generation.

Within the system of sub-models (mode choice, route choice etc.) represented by p , the linkages made by logsum variables are well established. It is natural to think that the linkage to the generation model could also be formed by a logsum. The properties of the logsum in measuring overall utility, which is effectively accessibility, make it natural to use in that context.

3.1 Basing trip generation on utility theory

It is required, therefore to find a formulation of the generation model that can accept an input of accessibility, in the form of a logsum, to give a completely linked model system.

In our previous work we showed how this could be achieved in the form of a model comprising two components, designed to take account of the difference in nature of the decision to make the first tour (or visit) for a given purpose in the day and the decision to make multiple tours:

- a model of the choice between making zero and one-or-more tours;
- a model of the choice of making at least one further tour, given that a positive number of tours has been made.

By using binary logit models for these choices it is possible to include the accessibility logsum in the utility of the more-travel alternative with confidence that the utility of each additional tour is being represented in a reasonable way.

Models of this type have been found to be successful in explaining generation in urban and regional contexts. In other contexts, such as long-distance and international corridors, simpler models have been found to be satisfactory, largely because the possibility of making two tours in a day, or even a week, is very unlikely.

An alternative approach is mentioned by Larsen (2003) who suggests that the parameter of a Poisson distribution can be interpreted as a utility. However, this suggestion is not yet convincing in his equations – it “may have some merit” as he says, but that still needs to be demonstrated. Essentially the mechanism by which accessibility influences trip rates needs to be clarified.

Thus the only plausible approach currently known for including accessibility in a proper model of generation is to set up that model as a choice between alternatives with explicit numbers of choices and to attach the logsum accessibility variable to these in a reasonable way. An example of this procedure is the ‘stop-go’ model discussed in our previous work.

3.2 Appropriate forms of model

In the stop-go model the choice of whether to travel at all (to make zero vs. one or more journeys) and a recurring choice of whether to make further journeys (1/2+, 2/3+ journeys etc.) are modelled as binary logit choices. If the probability of an individual n making at least one journey is p_n , and the probability of making a further journey at each stage is q_n , then the expected number of journeys is

$$D_n = \frac{p_n}{1 - q_n} \quad (1.1)$$

*When the 0/1+ model and the 1/2+, 2/3+... models are the same, the stop-go model reduces to a geometric model with parameter $(1 - p_n)$.*¹

In this case, suppose the ‘stop’ alternative for individual n has utility V_n^0 , which may incorporate all non-accessibility (e.g. socio-economic) effects, and the ‘go’ alternative has a utility of $\lambda \cdot V_n^*$, a multiple of the logsum. Then the probability of travel is:

$$p_n = q_n = \frac{\exp(\lambda \cdot V_n^*)}{\exp(\lambda \cdot V_n^*) + \exp(V_n^0)} \quad (1.2)$$

And equation (1.1) becomes

$$\begin{aligned}
 D_n &= \exp(\lambda \cdot V_n^* - V_n^0) \\
 &= \alpha \cdot \exp(\lambda \cdot V_n^*)
 \end{aligned}
 \tag{1.3}$$

where α does not depend on accessibility. That is, the exponential model has the same expectation of the forecast number of trips as a stop-go model in which the two model components are identical. Since for forecasting the expectation D is the chief statistic of interest, we may consider the exponential model to be an implementation for forecasting of this simplified stop-go or geometric model. The exponential model thus has a secure basis in utility theory.

It is interesting to compare this derivation of exponential trip generation with the Poisson model. Firstly note that, while the two models both give rise to a mean trip rate that is an exponential of the logsum, the probability distributions for the actual number of trips made by an individual are very different. Figure 1 shows geometric and Poisson distributions with means equal to one, four and ten. The mode of the Poisson distribution occurs around the mean, whereas the mode of the geometric distribution is always zero. The geometric distribution also has a larger variance. Visually, the difference between the distributions is clear. Visually, it is also clear that the difference is less for lower trip rates.

For urban and regional contexts, the acceptability of this special case of the stop-go model is questionable. Behaviourally, the decision whether to travel at all (0/1+ trips) is usually found to be quite different from the decision whether to make a further trip (1/2+, 2/3+ etc.), and so the geometric model cannot be recommended for trip generation in those contexts.

For long-distance travel, however, a single model is acceptable. In Daly (1997) it was argued that, for these contexts, the exponential model was an acceptable approximation to the logit model of binary choice between 0 and 1 tours. However, on the basis of the discussion above, we see that the exponential model is actually an exact implementation of the geometric model, of which each step is modelled by a binary choice. The fact that the probability of making more than one tour is very small is no longer important to the theory but is important to the ability of the model to represent the actual behaviour accurately.

Summarising, we may conclude that:

- the exponential model can be considered to be an implementation of the geometric model, which in turn is a simplification of the stop-go model described by Daly (1997);
- the exponential model thus has a secure basis in utility theory;
- the Poisson model can also be set up to generate a mean trip rate which is proportional to the logsum; however, the link to utility theory in this case has yet to be established;
- the problem with the exponential model is that it does not represent observed behaviour in urban and regional contexts well, hence the

- requirement to model 0/1+ choice separately from choices involving multiple tours;
- Larsen (2003) found a corresponding problem with the Poisson model in some of his tests on Norwegian data, where he found it necessary to introduce an initial binary choice model for the 0/1+ choice;
- in long-distance corridors there is little need to distinguish the 0/1 choice from the repeat choices, and the exponential model then becomes a much more reasonable prospect.

Finally, we should note that the exponential model is different from a constant-elasticity model. While the demand changes proportionally to stimulus changes in both models, in the exponential model the stimulus is an *absolute* change in utility, while in the constant-elasticity model the stimulus is a *proportional* change, usually in a generalised cost. The difficulty in the elasticity model is then to define a zero for generalised cost, i.e. defining exactly which components should and should not be included. This difficulty does not arise in the exponential model.

4. CONSUMER SURPLUS FOR THE EXPONENTIAL MODEL

We consider a model system comprising a compound model of trip generation and the choice, for each generated trip, of one of a number of alternatives. We refer to the choice model as a model of distribution; it may refer to mode, route or other choices.

An advantage in this context of an exponential trip generation model is the availability of an exact analytic expression for consumer surplus as a function of characteristics of any of the alternatives in the distribution model. This expression is valid whenever the trip *distribution* model is a GEV model: MNL, nested logit, cross-nested logit and other models are all encompassed.

The expression for consumer surplus is presented here, along with an explanation of its properties. The simple MNL case is illustrated first, before the general GEV formulation is considered.

4.1 Multinomial logit (MNL)

In an MNL choice model, the probability of choosing alternative i , for any individual n , is:

$$P_{in} = \frac{\exp(V_{in})}{\sum_j \exp(V_{jn})} \quad (1.4)$$

Where V_{in} is the observed (modelled) portion of utility for alternative i as experienced by individual n .

The logsum:

$$V_n^* = \log \sum_j \exp(V_{jn}) \quad (1.5)$$

is a measure of the individual's expected utility from the choice situation². The derivative of the logsum with respect to the utility of a particular alternative i is:

$$\frac{\partial V_n^*}{\partial V_{in}} = \frac{\frac{\partial}{\partial V_{in}} \sum_j \exp(V_{jn})}{\sum_j \exp(V_{jn})} = \frac{\exp(V_{in})}{\sum_j \exp(V_{jn})} = P_{in} \quad (1.6)$$

i.e. the derivative of the expected utility with respect to the utility of alternative i is equal to the choice probability of that alternative. This has an intuitive appeal. The choice probability of an alternative can be interpreted as the demand function for that alternative, given a single decision maker facing a single choice situation: in this context the expected utility is the consumer surplus for that individual.

With an exponential trip generation model, the overall trip rate for an individual is given by:

$$D_n = \alpha \cdot \exp(\lambda \cdot V_n^*) \quad (1.7)$$

where V_n^* is the accessibility logsum, as in (1.5). The demand function for alternative i is:

$$D_{in} = D_n \cdot P_{in} = \alpha \cdot \exp(\lambda \cdot V_n^*) \cdot \frac{\exp(V_{in})}{\sum_j \exp(V_{jn})} \quad (1.8)$$

The consumer surplus is usually defined as the area under, mathematically the integral of, the demand function. That is, it is the function whose derivative, with respect to the utility of each alternative, is the demand function for that alternative. This has the same intuitive basis as the link between expected utility and choice probability above. The consumer surplus function for individual n is:

$$CS_n = \frac{\alpha \cdot \exp(\lambda \cdot V_n^*)}{\lambda} - \frac{\alpha}{\lambda} \quad (1.9)$$

which is unique up to the addition of an integration constant. The term $-\frac{\alpha}{\lambda}$ is such a constant, since it does not depend on the utilities, and has been chosen because it ensures that the function is well behaved as $\lambda \rightarrow 0$.

The validity of the expression can be verified by differentiation with respect to V_{in} , remembering that

$$\frac{\partial V_n^*}{\partial V_{in}} = \frac{\exp(V_{in})}{\sum_j \exp(V_{jn})} = P_{in} \quad (1.10)$$

then

$$\begin{aligned} \frac{\partial CS_n}{\partial V_{in}} &= \frac{\partial CS_n}{\partial V_n^*} \cdot \frac{\partial V_n^*}{\partial V_{in}} \\ &= \left(\alpha \cdot \exp(\lambda \cdot V_n^*) \right) \cdot \left(\frac{\exp(V_{in})}{\sum_j \exp(V_{jn})} \right) \\ &= D_n \cdot P_{in} \\ &= D_{in} \end{aligned} \quad (1.11)$$

By expanding the exponential term in (1.9) as a Taylor series, it is possible to show that, thanks to the inclusion of the constant $-\frac{\alpha}{\lambda}$,

$$CS_n = \alpha \cdot V_n^* + O(\lambda) \quad (1.12)$$

So as $\lambda \rightarrow 0$, i.e. the trip rate becomes constant, the consumer surplus becomes simply the fixed trip rate multiplied by the expected utility or logsum for a single trip.

4.2 Generalised Extreme Value (GEV)

In the general GEV framework, a discrete choice model is defined by a function $G = G(y_1, \dots, y_r)$ where $y_i = \exp(V_{in})$ in our notation: V_{in} is the observed portion of utility for alternative i , as experienced by the individual n . There are four conditions on G , set out by McFadden (1978), which are sufficient to ensure that the choice model given by:

$$P_{in} = \frac{y_i G_i}{\mu G} \quad (2.1)$$

is a RUM model, where $G_i = \frac{\partial G}{\partial y_i}$ and $\mu > 0$ is such that G is homogeneous of degree μ in y_1, \dots, y_r .

The expected utility gained by the decision maker is given by:

$$\tilde{U}_n = \frac{\log G + \gamma}{\mu} \quad (2.2)$$

where γ is Euler's constant. The derivative of the expected utility with respect to V_{in} , the modelled utility of alternative i is:

$$\frac{\partial \tilde{U}_n}{\partial V_{in}} = \frac{\partial \tilde{U}_n}{\partial y_i} \cdot \frac{\partial y_i}{\partial V_{in}} = \frac{G_i}{\mu G} \cdot y_i = P_{in} \quad (2.3)$$

i.e. the derivative of the expected utility with respect to the utility of alternative i is equal to the choice probability of alternative i , as anticipated.

With exponential trip generation and a GEV distribution model, the demand for an alternative i is given by:

$$D_{in} = \alpha \cdot \exp(\lambda \cdot \tilde{U}_n) \cdot \frac{y_i G_i}{\mu G} \quad (2.4)$$

The corresponding consumer surplus function is:

$$CS_n = \frac{\alpha \cdot \exp(\lambda \cdot \tilde{U}_n)}{\lambda} - \frac{\alpha}{\lambda} \quad (2.5)$$

This can be verified by differentiation with respect to V_{in} , bearing in mind (2.3):

$$\begin{aligned} \frac{\partial CS_n}{\partial V_{in}} &= \frac{\partial CS_n}{\partial \tilde{U}_n} \cdot \frac{\partial \tilde{U}_n}{\partial V_{in}} \\ &= \left(\alpha \cdot \exp(\lambda \cdot \tilde{U}_n) \right) \cdot \left(\frac{y_i G_i}{\mu G} \right) \\ &= D_{in} \end{aligned} \quad (2.6)$$

Once again $-\frac{\alpha}{\lambda}$ is an integration constant which ensures that the function is well behaved as $\lambda \rightarrow 0$. By expanding the exponential term as a Taylor series, it is possible to show that

$$CS_n = \alpha \cdot \tilde{U}_n + O(\lambda) \quad (2.7)$$

So as $\lambda \rightarrow 0$ the consumer surplus becomes simply the fixed trip rate multiplied by the expected utility for a single trip.

4.3 Comparison with rule-of-a-half for changes in accessibility

The rule-of-a-half approximation is commonly used to estimate changes in consumer surplus arising out of a change in the utility of a particular mode, or more generally a change in accessibility, with a corresponding change in demand. It is a useful tool where an exact form for the consumer surplus function is not known. The rule-of-a-half formula gives:

$$\Delta CS \approx D^0 \cdot \Delta \tilde{U} + \frac{1}{2} \cdot \Delta D \cdot \Delta \tilde{U} \quad (3.1)$$

Here, $\Delta D = D^1 - D^0$ is the change in demand arising out of a change in accessibility/expected utility $\Delta \tilde{U} = \tilde{U}^1 - \tilde{U}^0$. The subscript n representing a particular individual or population segment has been dropped for the sake of clarity. The notation in (3.1) matches that in section 4.3 for a change in the expected utility from a GEV choice. The rule-of-a-half is often expressed for a single mode alternative i :

$$\Delta CS_i \approx D_i^0 \cdot \Delta V + \frac{1}{2} \cdot \Delta D_i \cdot \Delta V_i \quad (3.2)$$

and the total change in consumer surplus is calculated as the sum over all modes. In the case of a mode becoming unavailable, special techniques using multiple rule-of-a-half calculations must be applied (Nellthorp and Hyman, 2001).

Since the rule-of-a-half is an approximation, the exact form of the consumer function ought to be used when it is available; as shown here, the exact formula is in fact simpler. We compare the two expressions for the case of exponential trip generation. The exact expression for the change in consumer surplus arising out of a change in accessibility is:

$$\Delta CS = \frac{\alpha}{\lambda} \cdot (\exp(\lambda \tilde{U}^1) - \exp(\lambda \tilde{U}^0)) \quad (3.3)$$

which can be derived easily from (2.5). Note that:

$$\Delta CS = \frac{D^0}{\lambda} \cdot (\exp(\lambda \Delta \tilde{U}) - 1) \quad (3.4)$$

and

$$\lambda \cdot \Delta CS = \Delta D \quad (3.5)$$

a very simple calculation for use in practice. (3.4) can be expanded as a power series:

$$\Delta CS = D^0 \left(\Delta \tilde{U} + \frac{\lambda \cdot (\Delta \tilde{U})^2}{2!} + \frac{\lambda^2 \cdot (\Delta \tilde{U})^3}{3!} + \dots \right) \quad (3.6)$$

while the rule-of-a-half expression in (3.1) and the formula (3.5) imply:

$$\Delta CS^{ROH} = D^0 \cdot \Delta \tilde{U} + \frac{1}{2} \cdot \Delta \tilde{U} \cdot \lambda \cdot \Delta CS \quad (3.7)$$

and substituting (3.6) into (3.7):

$$\begin{aligned} \Delta CS^{ROH} &= D^0 \cdot \Delta \tilde{U} + \frac{1}{2} \cdot \Delta \tilde{U} \cdot \lambda \cdot D^0 \left(\Delta \tilde{U} + \frac{\lambda \cdot (\Delta \tilde{U})^2}{2!} + \frac{\lambda^2 \cdot (\Delta \tilde{U})^3}{3!} + \dots \right) \\ &= D^0 \left(\Delta \tilde{U} + \frac{\lambda \cdot (\Delta \tilde{U})^2}{2} + \frac{\lambda^2 \cdot (\Delta \tilde{U})^3}{2!2} + \frac{\lambda^3 \cdot (\Delta \tilde{U})^4}{3!2} + \dots \right) \end{aligned} \quad (3.8)$$

By subtracting (3.8) from (3.6) we can see that:

$$\Delta CS = \Delta CS^{ROH} + D^0 \left(\lambda^2 \cdot (\Delta \tilde{U})^3 \left(\frac{1}{3!} + \frac{1}{2!2} \right) + \lambda^3 \cdot (\Delta \tilde{U})^4 \left(\frac{1}{4!} + \frac{1}{3!2} \right) + \dots \right) \quad (3.9)$$

or, replacing the original expression for rule-of-a-half:

$$\Delta CS = D^0 \cdot \Delta \tilde{U} + \frac{1}{2} \cdot \Delta D \cdot \Delta \tilde{U} + D^0 \left(\lambda^2 \cdot (\Delta \tilde{U})^3 \left(\frac{1}{3!} + \frac{1}{2!2} \right) + \lambda^3 \cdot (\Delta \tilde{U})^4 \left(\frac{1}{4!} + \frac{1}{3!2} \right) + \dots \right) \quad (3.10)$$

This expression allows some insight into the exact consumer surplus expression, as follows.

- The first term gives the accessibility improvement for the initial demand.
- The second term gives rule-of-a-half consumer surplus applied to the new demand which is generated by the accessibility improvement.
- The remaining terms are small but all are negative (a linear approximation overstates the integral of the exponential function).
- When $\lambda \rightarrow 0$ the accessibility improvement is the only benefit (total demand remains constant).

Clearly the simple formula (3.5) would be used in practice, but this expansion reassures us that even when λ is small and poorly estimated the result will be well-behaved.

5. ESTIMATION OF TRIP GENERATION MODELS

In general, software for the estimation of the types of models discussed in this paper is available. Such packages as ALOGIT and STATA offer adequate facilities for logit modelling and Poisson regression.

The difficulty, however, is in finding appropriate data for the estimation. We may consider three types of data.

First, **Revealed Preference (RP)** data should be the basic source for model estimation whenever possible. However, the cheaper forms of RP data, such as counts and intercept surveys, are not very useful for modelling trip generation in urban and regional contexts, because the chance that trips are observed in such surveys depends not only on the probability that the trip is made, but on also the probability that it is of sufficient length to be observed in the survey. Attempting to estimate the total number of trips made is then subject to substantial error. Attention is then directed to home interview surveys, which are an excellent source for trip generation data, but which are expensive to conduct. In some cases reference to national data sources will be useful.

In long-distance studies the issues are different. Here we can usually obtain RP information on a substantial fraction of trip-making in the corridor and build a model that represents the frequency of travel in the corridor as a function of accessibility. Such models are often quite successful, in that they give a significant relationship between accessibility and trip rate. A potential problem, however, is that the models incorporate a large element of destination choice, i.e. the decision to travel in the corridor, along with the actual trip generation; this is not necessarily a serious issue. The other problem is that the model implicitly assumes that if accessibility is improved (for example), people will begin to behave like those who currently live nearer the corridor. Since residential location is often correlated with other socio-economic and often attitudinal variables, this assumption may be inaccurate. It is also necessary to have information on the total population that *might* have travelled in the corridor. Nevertheless, RP data should be used whenever possible.

Second, because of the nature of trip generation, that an explicit choice does not exist as an alternative to travelling, the use of **Stated Preference (SP)** data in its usual discrete choice form is difficult. Because making trips is inextricably connected with activity patterns, it is very difficult for SP respondents, particularly within the short time frame of the interview, to give preference between options that involve trip making and those that do not.

The third type of data that has been used in corridor contexts is **Stated Intentions (SI)** data. Here, travellers are asked how many more or fewer trips

they would make if accessibility changed. The success of models built on this type of data is perhaps surprisingly good, as we sometimes find a number of plausible significant variables and reasonable correlation with models built on RP data. Effects such as greater response to losses in accessibility than to gains are found, which in themselves are not surprising but require judgement as to whether they represent short-term or long-term effects (e.g. Kouwenhoven *et al.*, 2006).

In each particular study context, analysts will need to construct a data base for model estimation that is specific to the issues of the study and the data that is available or can be collected.

6. CONCLUSIONS

There is a wide gap between the conventional methods of trip generation and the most sophisticated activity modelling methods. This paper attempts to improve standard modelling methods by improving the way in which data is analysed and connecting that better with the models applied in forecasting studies.

Utility maximisation is currently the best approach to setting up systems of forecasting models and we therefore seek to integrate generation models into a complete system, using the key variable of accessibility formulated as the overall utility or logsum from the other models of the system. At present this is possible only for generation models formulated as choice models.

For urban and regional models, it is usually found that the probability of making one or more trips has to be modelled separately from the probabilities of making multiple trips. However, for long-distance modelling that is not the case and a single model suffices. When a single model can be used, or as an approximation in other cases, the exponential model form can be used.

Consumer surplus can be calculated for model systems including an exponential frequency model by a very simple formula, which is close to the rule-of-a-half calculation when generation rates are small. This calculation is valid whenever the other travel choices in the system are represented by a GEV model.

Data issues in modelling generation are difficult. For urban and regional studies, home interview data, perhaps from national sources should be used wherever possible. For long-distance travel, intercept surveys will give the best basis. Stated Intentions data can sometimes be a valuable source.

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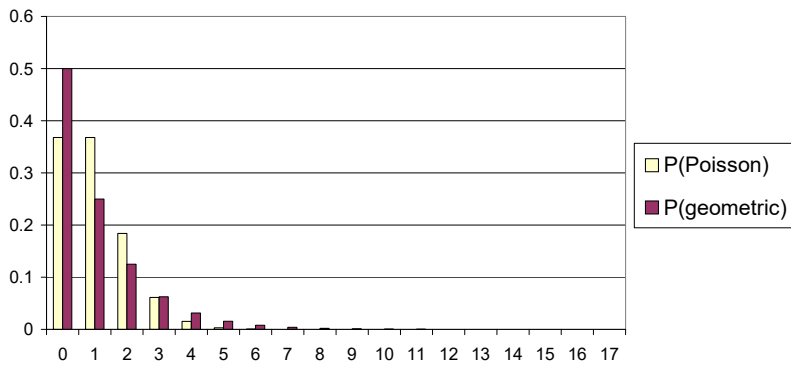
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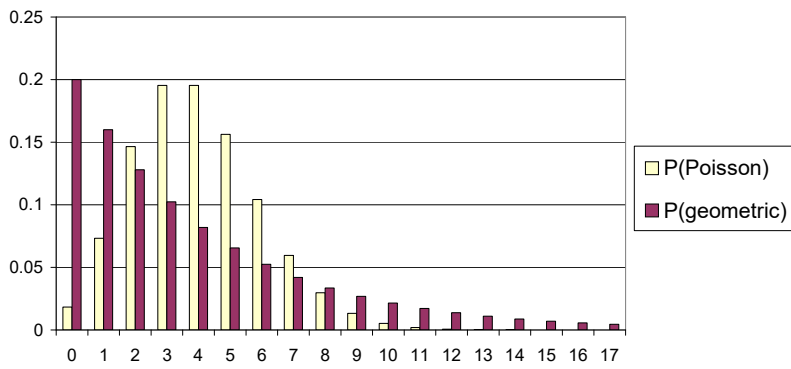
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Figure 1: Poisson and geometric distributions

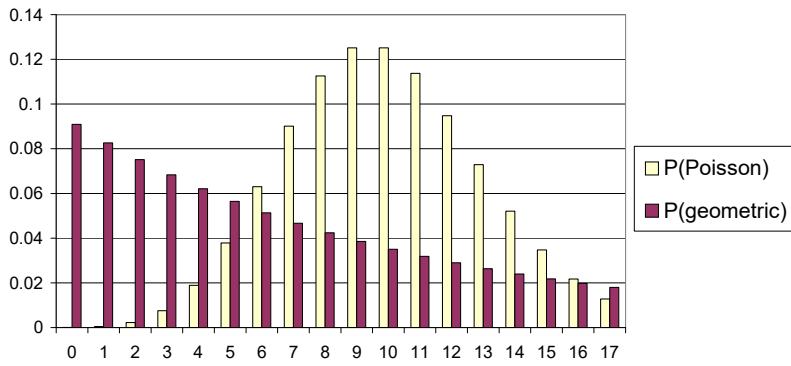
Mean = 1



Mean = 4



Mean = 10



Notes

¹ The geometric distribution is traditionally described as explaining the number of independent trials that take place until one 'success' is observed, and the parameter is the probability of success in any trial.

² Strictly, the expected utility includes a constant term equal to Euler's constant γ . However, this can be ignored as it is constant.