THE USE OF SCHEDULE-BASED ASSIGNMENTS IN PUBLIC TRANSPORT MODELLING

Andrew Daly
Hague Consulting Group

Traditionally, public transport assignments have been based on the assumption of a 'steady state' provision of service throughout the period being modelled, i.e. with the assumption that an *average frequency* for each public transport line gives an adequate representation of the service. Recently, modellers have more and more begun to question this assumption and modern computer power has given them the opportunity to make assignments based on the actual timetable or schedule of operations. The use of the schedule as a basis for assignment opens the possibility of improving the accuracy of the representation of the services in a number of respects and for this reason a number of computer packages are beginning to offer the facility of schedule-based assignment. Terms such as 'deterministic' or 'real-time' assignment are also used to describe procedures of this type.

However, the use of schedule-based assignment is certainly more costly in terms of computer time and the requirements for data input can be prohibitive. The intention of this paper is to discuss the methods used for schedule-based assignment and the advantages they offer, which depend very much on the context in which it is to be applied, to allow a more rational assessment to be made of the appropriate method for making an assignment in given circumstances.

It should be noted that schedule-based assignments can be applied to quite a range of transport modes: buses, trains etc. but also aircraft and ferries, in fact any modes that run to a specified schedule, can usefully be treated in this way. In the paper, reference will be made to 'public transport' to refer to all such modes.

1. SCHEDULE-BASED ASSIGNMENTS

The key aspect of a schedule-based assignment is that the basic description of the network is given in terms of vehicle runs rather than public transport lines. Simply, the level of detail is increased. Of itself, this increased detail – if correct – will generally improve the quality of modelling; whether this quality improvement will be sufficient to justify the additional effort of modelling vehicle runs is one of the issues that this paper is intended to clarify.

The additional effort of modelling vehicle runs is quite considerable. The volume of data is much larger than that required for a frequency-based assignment, since the scheduled time at each station has to be recorded, most importantly to determine whether particular interchanges are or are not feasible. To take full advantage of the additional detail offered by the schedule-based assignment it is also desirable that differences in running time between the various services are recorded, while small differences in link times would typically be neglected in a frequency-based assignment. To obtain and check all of this detailed information from a transport operator's timetable or operating schedule is obviously a considerable effort.

Further, particularly when testing alternative network designs for the future, it is often the case that the operator itself is unable to prepare detailed running schedules for a wide range of network designs. This means that either schedule-based assignments have to be abandoned or that approximations have to be made. However, the use of approximations for such forecasts can introduce systematic inconsistencies relative to forecasts in which schedules are available.

As in any assignment procedure, the main steps in making a schedule-based assignment are the generation of alternatives and the splitting of passenger flows among them. In this case, the alternatives would include not only paths but also vehicle runs. Generally, it can be said that the generation of alternatives is primarily a task of network processing, where the difficulties lie in efficient analysis of a large and complicated task, while the prediction of the split is a problem of modelling behaviour. In the present paper, the interest is primarily in this second issue but it is useful first to consider the ways in which the alternatives can be generated.

A number of software packages have been developed to deal with schedule-based assignment. As in most cases in which new methods are implemented, the first operational systems have been written as *ad hoc* programs by researchers and it is only recently that schedule-based assignment has been available in a major commercial package (EMME/2). While grafting schedule-based assignment onto an existing package has some disadvantages in terms of its flexibility and probably its run speed, compared to an *ad hoc* package, there are also numerous advantages and it is to be expected that other software manufacturers will offer this possibility in the future, should schedule-based assignment continue to grow in popularity.

The procedure implemented in EMME/2 has been described by Florian (1998). The algorithm appears to work as an extension of the basic EMME/2 procedure, which would generate a very large number of possible alternatives were not a pruning implemented which is based on the 'dominance' of one alternative over another. Dominance is under the control of user-specified parameters but can be set up, for example, to eliminate all alternatives that take a traveller to a point later than his best alternative to that point. This definition leads to a reduced set of alternatives such that a traveller departing at any given moment has a unique path to each destination. The advantage of using a comprehensive commercial package as a basis for the assignment is seen in the facilities that are available in EMME/2 for defining services and the flexibility of the parameters that can be specified for path choice.

A similar approach can be found in the Prolop package used by Netherlands Railways (Dam and Kieft, 1996). Here the numbers of alternatives are restricted by *ad hoc* rules such as specifying that interchanges may not take place at stations served by a single train service. Nine such criteria, some of them involving user-adjustable parameters, are employed. This leads to a reduced set of alternatives; comparing the criteria with those used in EMME/2, it appears that the choice sets in Prolop may be somewhat larger, since more interchange possibilities are retained.

Rather different is the procedure used by Nielsen *et al*. (1999) and Petersen (1999). Here the entire system, both path selection and the splitting of passengers, is based on sampling along the lines of the Stochastic User Equilibrium methods familiar in highway assignments. Given a draw of a random departure time and a random perturbation of the network, a traveller departing at a specific time is assigned to the unique least generalised cost path. Repeated draws yield different departure times but also potentially different paths for any given departure time. Attractive features of this method are that the path choice method remains simple, since only one path needs to be found for each random draw, and that the method can also accommodate a much more sophisticated description of the travelling population, with path choice parameters (such as 'values of time') varying continuously within the population. Disadvantages are that information has to be available about the variation of the path choice parameters and that the run times can become large when the networks are large because of the need for large-scale sampling.

An important common feature of all three of these approaches is the intimate relationship between the time of departure and the path chosen. In most cases, only one path is modelled for each time of departure.

The three computer implementations of schedule-based assignment are certainly not the only implementations currently in use, indicating that the interest in schedule-based assignment is quite widespread. It is also likely that further implementations will be made, particularly by commercial software manufacturers.

2. CHOICE OF SERVICE

In the previous section attention was given to the process of generating paths and various methods for discarding or 'pruning' less efficient paths were described. Given that efficient paths can be generated for a given departure time, the issue of path choice is then closely linked with that of predicting departure time. Two issues arise in this context:

- the underlying preference of the travellers for travel at different times; and
- the preference they have for services which may vary in quality at different times.

These two issues are discussed in turn.

2.1 Preferences for Travel Times

Usually, assignments are made for a specific time period, such as the morning peak, and an assumption is effectively made that the level of demand is constant within that period. When that period is at all extended, e.g. it exceeds an hour, such an assumption is no longer very accurate. For a frequency-based assignment, the failure of the assumption need not be very important because the output is only ever considered to be an average for the period. However, for a schedule-based assignment, when interest focuses on the numbers of trips carried by each vehicle run, a better procedure seems to be needed for spreading travellers over runs.

The central problem here is of supply-demand interaction. Travellers choose a time for their trip as a function of the transport supply as well as of their basic requirements. There is no point turning up for a bus at 8.10, if that is the time you wish to travel, when you know that buses run at 8.00 and 8.30. This type of behaviour has been studied in more detail in the context of car travel, where drivers adjust their behaviour to avoid congestion (see, for example, van Vuren *et al*., 1998), but even in that context no solution has yet been found to determining when drivers would really like to travel. Interviewing procedures could be developed to address the issue, but these do not yet seem to have been carried out on a large scale.

In this context, the only reasonable procedure is to assume that the underlying demand for travel is distributed approximately uniformly over the period in question. Given sufficient data, the pattern of demand can in principle be modelled more accurately by reducing the length of the period, but it is important to avoid interpreting a choice of services motivated by supply as being an underlying preference. For this reason, in the absence of further information about preferences, the length of the periods being studied should be sufficient to include a whole cycle of the timetable.

The difficulty of obtaining good information about passengers' preferences for times of travel is a significant limitation on the improvement that can be obtained from schedule-based assignment. It is to be hoped that future research will reduce this limitation, possibly learning from the more extensive research on highway assignment.

2.2 Quality of Alternative Services

Classical behavioural modelling in transport planning measures the quality of service offered by travel alternatives by generalised cost, a measure of the 'impedance' to travel by that alternative including the cost of the alternative and the time taken – often separated into components such as walking, waiting, 'in-vehicle' etc. – weighted together to give an overall function. It is natural to consider whether such measures also could be used to compare alternative services. It is helpful to separate two separate aspects of each alternative, its specific generalised cost and its competitive situation relative to other services.

Each alternative can be considered to have a specific generalised cost, which is made up of:

- cost, i.e. the fare to be paid, often not varying with the path chosen although this can become an important issue when operators compete on price as well as on quality;

- travel time separated into time spent in vehicles of different quality, e.g. intercity trains, slow trains, trams, buses etc., in car or walk (or cycle) access and in any walking at interchanges; account may also be taken in more sophisticated software of other aspects of comfort, in particular of crowding and the class of travel but perhaps also of further comfort variables;

- number of interchanges (transfers) and interchange time; and

- waiting time and/or a measure of the distortion caused to the traveller's preferred schedule.

These specific components will typically appear as terms in the generalised cost of the alternatives. Weights for the components can be derived by statistical estimation procedures from observations of passengers' choices, or can be specified on the basis of previous experience.

The cost, travel time and interchange components are usually simple enough to formulate and estimate from network models. However, a more difficult problem is to describe waiting time and its overlap with the inconvenience caused to the traveller by diverting from his preferred schedule, which is often called 'schedule delay', even when the impact is actually to cause an earlier departure than desired.

A concept that is used in the work described by Nielsen *et al*. (1999) is that of 'hidden waiting time'. The idea here is to note that the average difference between the preferred time of departure and the actual time of departure of the first service used is half the headway of that service (providing the service is regular). However, a traveller who knows the timetable and is able to control his actual departure time will not spend all of that time waiting at the station, he or she will spend part of it at home (or wherever) under more attractive circumstances. This 'hidden waiting time' can be estimated and appropriate estimates of impedance can be estimated from observations of travellers' behaviour. The random sampling assignment procedure used in the Nielsen work also makes it possible to link the estimates of hidden waiting time with the possible choice of different paths at different travel times. Other software packages such as EMME/2 also have facilities to make similar calculations when suitable data is available.

It is not the case, however, that all passengers know the timetable and can control their departure times. These assumptions would be appropriate for travel ***to*** work by lower-frequency services, but in other circumstances one or other of the assumptions would be dubious. Estimating the impedance weight of hidden waiting time from observations allows some account to be taken of the proportion of the population for which the assumptions are true, but this approach gives a rather broad average that does not depend on the local situation and the scheduling of the specific services used. The procedure also ignores any difference between the actual arrival time and the desired arrival time.

To exploit fully the potential of schedule-based assignment it is therefore necessary to consider in more detail the local competition between services with which the passenger is confronted.

2.3 Competition between Services

For each alternative a <u>measure of its competitive situation</u> is required. This has to do with its scheduling relative to the other alternatives. Thus if two buses are scheduled to run closely together with little difference in their intrinsic quality, the first bus will pick up many more passengers than the second, simply because it arrives first at the stops where passengers are waiting.

Both the competitive situation and the schedule delay / waiting time are affected by variables that have to do with frequency, waiting time, scheduling within the hour etc.. Exactly how these variables should be specified is not clear: one approach is described here that is based on the concept that the waiting time and schedule variables must be specified in the models in ways that help to explain the ways in which travellers choose trains. This approach has been developed for application in the Prolop model mentioned above (Dam and Kieft, 1996) and its associated demand forecasting model ProMiSe (Cohn *et al*., 1996). The approach is based on the concept that the travelling population is divided into a number of groups with different modes of behaviour. Of course we do not know the relative distribution of these behavioural groups in the population, nor would it be easy to get this distribution from a questionnaire.

Each of three groups is considered in turn. Travel time and interchange variables also form part of the generalised cost; these are assumed to be the same for all groups. For the consideration of the behaviour of these groups, it is assumed that the preferred travel times for each group are distributed uniformly over the time period under consideration.

Group 1. Don't know timetable

These people just go down to the station and get on the next service. They have no schedule delay, but experience a generalised cost that is equal to the waiting time before the service departs. The number of group 1 people on each service is proportional to the interval before each service.

Group 2. Departure constrained

These people do know the timetable, but cannot adjust their departure time. They experience a schedule delay that is equal to the interval before the service – they can use this time, perhaps, for some activity that is not highly preferred but is better than waiting, this is the 'hidden waiting time'. The number of them on a given service is proportional to the interval before each service.

Group 3. Arrival constrained

These people know the timetable and adjust their departure to get an arrival as close as possible before their appointment. They experience a delay at the arrival end proportional to the interval after their service arrives. The number of group 3 people on each service is proportional to the interval ***after the service arrives*** at the destination.

The behaviour described above can be summarised in the following Table.

Group	Behaviour	Generalised Cost	No. of Passengers
1	Turn-up-and-go	½ . interval1 . wt	$\propto f_1$ * interval1
2	Departure constrained	½ . interval1 . dl1	$\propto f_2$ * interval1
3	Arrival constrained	½ . interval2 . dl2	$\propto f_3$ * interval2

interval1 is the interval *before* the departure of the service chosen from the departure of the previous service;

interval2	is the interval *after* the arrival of the service chosen until the arrival of the following service;
wt	is the (generalised) cost per minute of waiting at the station;
dl1	is the cost per minute of having a service leave after you want;
dl2	is the cost per minute of having a service arrive before you want; and
f_1, f_2 and f_3	are the fractions of the population exhibiting each mode of behaviour.

The generalised cost of each service thus requires additional components which have an average value in the population equal to

$$\beta_1 \,.\, \text{interval1} + \beta_2 \,.\, \text{interval2}$$

with

$$\beta_1 = \tfrac{1}{2} \,.\, (\text{wt} \,.\, f_1 + \text{dl1} \,.\, f_2)$$
$$\beta_2 = \tfrac{1}{2} \,.\, \text{dl2} \,.\, f_3$$

It is also necessary to include in the model the intrinsic attraction of the services resulting from the intervals before departure and after arrival, i.e. the mechanism that gives rise to the numbers choosing each alternative stated in the Table. For the work on the Prolop/ProMiSe system, logit models were used and in these models it is possible to arrange that the share of an alternative is proportional to a specific variable by the use of the ***logarithm*** of that variable in the generalised cost function (Daly, 1982). Again taking the average of the three behavioural groups the terms in the generalised cost function need to be

$$-\beta_3 \,.\, \log \text{interval1} - \beta_4 \,.\, \log \text{interval2}$$

with

$$\beta_3 = f_1 + f_3$$
$$\beta_4 = f_2$$

In deriving estimates of the β coefficients there is a problem that the variables interval1 and interval2 are strongly correlated with their logarithms, so that large amounts of data are necessary to derive separate estimates. With limited data, it will be necessary to make a choice of estimating *either* β_1 and β_2 *or* β_3 and β_4.

Further types of behaviour can be imagined, of course, but in many cases the generalised cost function extensions set out above will give good approximations to their behaviour too. It is also necessary to keep the model reasonably simple.

For the Prolop/ProMiSe system, models of the form above were estimated from observations of actual train usage. Note that when personal characteristics and travel purpose do not form part of the model specification, the model can be estimated very simply from counts of travellers using competing services. Of course it is interesting to know how these personal characteristics affect behaviour, but to investigate these influences interview data would be needed. Because of these data limitations, β_3 and β_4 only were estimated along with the time components of the generalised cost functions. The fare structure used by Netherlands Railways does not discriminate between train types and it was therefore not possible to estimate a coefficient for a fare variable.

The results obtained (given by Dam and Kieft, 1996) are shown in the Table below; however, subsequently further data was collected and improved models have been estimated for the current implementations of the system.

Estimated Service Quality Coefficients

Coeff.	Variable	Estimate	Standard Error	Relative Value
	Intercity travel time	0.0270	0.0055	1.00
	Slow train travel time	0.0382	0.0053	1.41
	Interchange time	0.2 * no. ints.	*	3.08
	Number of interchanges	0.417	0.025	15.44
$-\beta_3$	log (interval1)	0.721	0.047	
$-\beta_4$	log (interval2)	0.186	0.043	

* because of the high correlation between interchange time and the number of interchanges, the assumption was made in this model that 5 minutes of interchange time were equivalent in generalised cost to one interchange.

The estimates made are highly plausible in their relative values and are also statistically significantly different from zero. The model explains 12.9% of the variance in choice of service (McFadden's ρ^2) which may be considered satisfactory given the limited nature of the data. Clearly models of this type can be improved by further work and/or better data and, as noted, an improvement has already taken place for the Prolop/ProMiSe system.

In particular, it is possible to add variables to the model. This is of great interest when modelling the competition between competing operators who offer different packages of price, comfort, reliability and speed for services between the same pairs of stations. When suitable data exists, either from user surveys or from stated preference or other interview work, the models can be extended quite simply to take account of further variables in addition to those incorporated to date in the Prolop/ProMiSe system. The model could be further extended to represent choice of class of travel, where this is offered by one or more operators.

3. QUALITY OF PUBLIC TRANSPORT SERVICE

In addition to the modelling of choice between alternative paths and services, it is often necessary to measure the quality of the entire public transport system for a journey, for example as input to a mode choice model. When a schedule-based assignment is used, it is necessary to take account of the various alternatives that are used by passengers in order to determine this 'composite' generalised cost. Three alternative specifications can be considered.

The first specification, effectively the method used by most frequency-based assignments, is to define the composite cost to be the generalised cost on the best alternative path. In a schedule-based assignment the equivalent would be the cost of the best vehicle run. An apparently obvious problem is that this method seems to overstate the quality of the public transport system; however, this is not usually serious because the mode choice model can correct for this. More serious in reality is that the additional benefit of services other than the best cannot easily be assessed. A service equally as good as the best clearly gives an advantage in increased frequency, but one that is a few minutes worse should not be considered in the same way. Ultimately an arbitrary criterion is usually adopted, counting services that are within a small amount of the best service as contributing to increased frequency and neglecting the others.

A second specification, used by programs such as EMME/2, is to define the composite cost as the average cost of the competing alternatives, weighted by the fraction of travellers that choose each of the alternatives. Again there are theoretical problems with this approach, in this case arising because of the potential inconsistency between the composite cost measure and the ***choice*** by passengers among the vehicle run alternatives. The inconsistency takes the form that a generalised cost reduction for one of the services can lead to an ***increase*** in the composite cost, obviously giving the potential for nonsensical results. When the model of choice over vehicle runs is of the logit form, it can be shown (see Appendix) that the average fails as a composite cost measure whenever there are alternatives substantially worse than the best; the average can be used in as a composite cost in these circumstances only as an approximation, fixing the choice probabilities to their base values, for example. For other model forms similar problems can arise, whether these affect the specific models employed in EMME/2 is not clear. A model based on sampling, such as the Nielsen *et al.* (1999) work, could escape from this inconsistency problem provided the generalised cost used in the mode choice model was the same as that used in the vehicle run choice model.

Theoretical problems can be avoided, when the vehicle run choice model is of the logit form, by using the 'logsum' composite cost (see Appendix) which can be proved to give consistent behaviour in all cases. However, even here there are some problems to be considered, which are more practical in nature. These concern the procedure used to estimate the coefficients of the generalised cost functions to be used in the mode choice and vehicle run choice models.

As was noted in the previous section, the coefficients of a vehicle run choice model can be estimated from quite simple data, perhaps just simple counts of passengers on competing services. However, the logsum from such a model cannot be used directly as the composite cost measure. An obvious defect is that variables may be omitted, for example in the Netherlands Railways case there is no competition on fares, fares do not therefore appear in the service choice model but they obviously do need to be incorporated in a mode choice model. Further, the variables describing frequency appear in the wrong form in this logsum: the greater the interval before the departure of a service, the more likely it is to attract passengers, so the lower the frequency the lower the ‘cost’. In the table of the previous Section, the coefficients of the variables relating to headways have a ***negative*** sign in the generalised cost, which is incorrect for mode choice modelling although correct in the service choice model. Obviously, for mode choice purposes it is necessary to arrange that lower-frequency services have a higher generalised cost.

The fundamental problem here is that a model predicting only choice among vehicle runs gives a logsum which could differ by an arbitrary amount from the correct generalised cost, since that arbitrary amount can be added to the generalised cost of each of the vehicle runs without changing the choice among them. The correct procedure to solve the problem is to estimate the model of choice among vehicle runs ***simultaneously*** with the mode choice model. In this way the coefficients of the generalised cost components can take account of both the choice among vehicle runs and the mode choice issues. However, this type of simultaneous modelling with disparate data sets presents problems of computer processing which can be addressed only by very specialised software.

The calculation of a composite cost measure thus presents a number of difficulties, whichever measure is adopted. Some steps can be taken to reduce these difficulties and analysts need to be aware of the problems and solutions when deriving mode choice and other models on schedule-based assignments.

4. SUMMARY and ASSESSMENT OF METHODS

In deciding whether to choose a schedule-based public transport assignment and, if so, which one, the analyst has a number of options, several of which have been reviewed in this paper. It appears that schedule-based assignments are growing in popularity and that more options will be available in the future.

Several methods have been used in practice to generate the path and vehicle run alternatives among which passengers are modelled as choosing. While it is not clear which method is best, it is clear that the very large number of potential alternatives needs to be kept under control. Major commercial packages can play a role here, while sampling approaches also offer considerable potential.

Travellers’ underlying preferences for different departure times are not well understood, this presents a limitation on the applicability of schedule-based assignment.

The key variables influencing passengers’ choices of vehicle runs can be formulated in generalised cost functions, although these need to be extended from standard forms to take account of scheduling interactions. The coefficients of these functions can be estimated from relatively simple observations. Such a model can be extended to take account of operator competition on price and quality.

Developing composite cost measures summarising the quality of the public transport system over a number of vehicle runs presents some difficulties. The ‘logsum’ measure is best theoretically but does not solve all of the problems. Sampling approaches are an alternative here but require consistency to be imposed throughout the model.

The choice of assignment method for any given study requires a balanced judgement to be made. Clearly, low reliability and high frequency of services (e.g. buses in central London) would argue against the use of schedule-based assignments, while low frequencies and a reliance on connections would give the opposite indication. Long-distance and international studies would appear to benefit most from these procedures.

The use of sampling procedures, taking advantage of increasing computer power, is an interesting approach, which also allows more sophisticated descriptions of traveller behaviour, such as distributions of preferences, to be incorporated. However, not all of the problems of these approaches have yet been solved.

In summary, schedule-based assignment appears to offer potential for improving public transport assignment methods. However, clarity about the circumstances under which this method should be applied requires further research and, in particular, practical tests in comparison with frequency-based assignments.

References

N. D. Cohn, C. L. Rohr, A. J. Daly, A. Dam, W. Oosterwijk and T. van der Star, 'ProMiSe: Policy-Sensitive Rail Passenger Forecasting for the Netherlands Railways', presented to *PTRC European Transport Forum,* 1996.

A. Dam and S. Kieft, *Toedelingsalgoritme Prolop-model*, Report, NSR Marketing-onderzoek & Advies, 1996 (in Dutch).

A. J. Daly, 'Estimating Choice Models Containing Attraction Variables', *Transportation Research Vol. 16B*, 1, 1982.

M. Florian, 'Deterministic Time Table Transit Assignment', presented to *PTRC Seminar on National Models*, Stockholm, December 1998.

O. A. Nielsen, G. Jovicic and J. M. Petersen, 'A large scale stochastic assignment model for passengers choices under delay and capacity considerations', being presented to PTRC/AET Conference, 1999.

J. M. Petersen 'Assignment model for timetable based systems (TPSchedule)', being presented to PTRC/AET Conference, 1999.

T. van Vuren, G. Hyman and A. J. Daly, 'Modelling Departure Time Choice', presented to *Colloquium Vervoersplanologisch Speurwerk*, The Hague, 1998.

Appendix: Average as Composite Cost in Logit Models

Suppose the composite cost measure V_S for the alternatives in a set S is defined by the average generalised cost:

$$V_S = \Sigma_{k \in S} V_k \, . \, p_k \, ,$$

where V, p give respectively the generalised cost and choice probability of each of the alternatives.

Then for *any* choice model we can obtain the composite cost derivative

$$\partial V_S / \partial V_c = \Sigma_k \, \partial p_k / \partial V_c \, . \, V_k \; + \; p_c$$

If the model is *logit*,

$$\log p_c \; = \; V_c \; - \; \text{logsum}, \qquad \text{where} \;\; \text{logsum} \; = \; \log \Sigma_{k \in S} \exp V_k$$

then the derivatives of the choice probabilities are

$$\partial p_k / \partial V_c = (\, \delta_{kc} - \Sigma_j \, \delta_{jc} \, . \, p_j \,) \, p_k = (\, \delta_{kc} - p_c \,) \, p_k \;\; ,$$

using the usual notation that δ_{kc}=1 if k=c and δ_{kc}=0 otherwise. Substituting in the composite cost derivative we can then derive

$$\partial V_S / \partial V_c = p_c \, (\, 1 + V_c - V_S \,)$$

$$< 0 \text{ , if } V_c \text{ more than 1 unit worse than } V_S.$$

This derivative has to be positive, the composite cost must change in the ***same*** direction when the generalised cost of one of the alternatives changes, so we conclude that the average as composite fails obviously whenever there is a change in an alternative whose cost is more than one unit worse than the average.

Thus if the average is to be used as a measure of composite cost, the choice probabilities must be fixed at the initial values, implying that changes in the average can be used as an approximation to changes in the 'logsum' value – theoretically the correct value for logit models – only when the changes relative to the initial probabilities are small.

The difference between logsum and average can be calculated by

$$V_S \; - \; \text{logsum} \; = \; \Sigma_{k \in S} \, p_k \log p_k.$$

That is, the logsum is always greater than the average (since log p < 0) by an amount that depends on the number of alternatives and their market shares.