ESTIMATING VALUES OF TRAVEL TIME[1]

Andrew Daly
Hague Consulting Group

The objective of this paper is to set out the methodological development that led to the adoption of the modelling method used in most current estimations of the value of travel time, to explain the main principles of the method and to show how a number of current issues are handled in that framework.

The paper is a summary of previous work and makes no claim to originality. Moreover, time and space are not sufficient to make a full review of the extensive literature on this subject. The objective is thus to indicate the development of the modern methods with reference to a number of illustrative studies that have been made. This will inevitably lead to my giving undue prominence to my own work and that of my associates, for which I apologise.

1. INTRODUCTION

The 'value of time' has a commonsense interpretation in everyday English, albeit somewhat vague. However, for precise estimation it is necessary to refine that definition in terms of an acceptable theory of human behaviour: the only theory sufficiently developed at present for this purpose is that of micro-economics. The issues arising in defining values of time are discussed in the following sections to define the topic of the paper and are followed by a statement of the structure of the remainder of the text.

1.1 Commonsense Interpretation of Value of Time

The notion that time can have a value is implicit in a number of constructions in the English language: we speak of spending or saving time, for example. Moreover, it is clear that travellers and other consumers are willing, to some extent, to spend money on faster modes of travel, labour-saving devices etc., whose main objective is to save time. The notion that time has some characteristics akin to commodities that can be bought and sold is thus implicit in the language and observable in everyday life.

However, there are important differences between time and real commodities. Each one of us has exactly 24 hours to spend every day and there is no way in which time can be transferred between days: it cannot be bought from another person, borrowed or stockpiled. What is then really meant when we speak of 'saving' time is that time is **transferred** between activities, e.g. from travel to some other, presumably preferable, activity. The 'value of travel time' can then only be interpreted as the

---

[1] The paper was prepared for the Course on Value of Time at Easthampstead Park on 28 October 1996.

value of being able to transfer time from travel to the traveller's preferred alternative activity[2].

Observations of choice of faster modes of transport, purchases of labour-saving devices, or alternatively the failure to make such purchases, can then be interpreted as travellers or consumers making 'trade-off' decisions in which the value of money is compared with the value of transferring time between activities. This trade-off or 'willingness to pay' is the essence of what is meant by the value of time and, as will be seen below, the basis of the less formal methods of estimating time values.

The key motivation for requiring estimates of values of time is in 'appraising' highway construction or other transport proposals. By appraisal is meant the prior assessment of the costs and benefits of the scheme, with a view to determining whether or not it should be constructed or whether it has a higher priority than those of other meritorious proposals. A number of other issues, such as environmental impacts, are clearly relevant to such decisions but it is travel time savings[3] that are the major objective of many, if not most, transport schemes. The evaluation in monetary terms of those savings is then an important part of the appraisal process. Evaluation procedures are beyond the scope of the current paper and are covered in other papers presented at the Conference. However, knowledge of travellers' willingness to pay for time savings is a key component of those evaluation procedures. The procedures described in this paper are intended to estimate the willingness to pay for time savings.

1.2 Micro-Economic Definition of Time Value

A more precise definition of the value of travel time is given by Jara-Díaz (1996b). In that paper, two relevant definitions are discussed:

SVT the 'subjective value of time', the amount that an individual is prepared to pay to transfer time from travel to an (indeterminate) alternative activity; and

SPT the 'social price of time', the amount **society** is willing to pay to transfer its members' time from travel to other, presumably more productive, activities.

Jara-Díaz shows that the SPT differs from the SVT in ways that are essentially political in character, reflecting the different importance given to different groups, particularly different income groups, by the collective political decision-making process. In estimation the interest is in the SVT: this is what it meant by 'VOT' in the present paper.

Jara-Díaz sets up a framework within which this value can be given a meaning, modelling consumer choice as a utility maximisation under time and money constraint and including a discrete choice element. A more detailed historical exposition of the

---

2 The phrase 'value of time' (or VOT) is however used subsequently because of its convenience and wide usage.

3 Time 'savings' in this context should not necessarily be seen relative to the current situation but relative to an alternative future situation, e.g. 'do minimum'. That is 'savings' may actually relate to a situation that is worse than the present, but where the deterioration is less bad than it might have been.

background literature supporting this optimisation framework is given in another paper by Jara-Díaz (1996a). The key element derived from this framework is that each of the discrete alternatives k is assigned an indirect truncated[4] utility function U_k, which is the utility accruing to the consumer if choice k is made. U_k is thus the maximum value of the utility U, subject to the constraint that discrete alternative k is chosen.

It is important to the general understanding of time valuation that the indirect utility function defined in the optimisation theory can be associated with a commonsense notion of the utility as the general attractiveness of an alternative. Unlike the full utility function of micro-economics, the indirect utility thus contains characteristics that are associated in common sense with the attractiveness of the alternative, such as its price (which is **not** a variable in the full utility function U) as well as quality and other measures. It is this identity of common sense and economic theory that allows time value estimation to be given a proper economic interpretation. Simply, the consumer (traveller) chooses that alternative with the highest (indirect) utility.

Within the optimisation framework of Jara-Díaz, the SVT is defined to be (my notation)

$$\nu_k = (\partial U_k / \partial t_k) \; / \; (\partial U_k / \partial c_k)$$

giving the value of the time t_k spent in using alternative k as the ratio of the partial derivatives of the indirect utility U_k of that alternative with respect to its time t_k and cost c_k.

A further important point emerging from the theoretical analysis is that

$$- \partial U_k / \partial c_k = \partial U_k / \partial I = \text{constant (with respect to k)}$$

where I is the consumer's income and the constant can be interpreted as the marginal utility of money. This means that the indirect utility function has to be linear in the cost of the alternatives, with the same coefficient for all alternatives, an important limitation on the functional specification of the models that can be used.

The alternatives appearing in the model can refer to any mutually exclusive set from which a consumer chooses. A typical example in the travel context, to which the remainder of this paper is addressed, is the choice of mode, but route choices and other 'within mode' choices are also often used for VOT analyses.

The micro-economics is based on a theory of marginal changes. Thus any conclusions drawn from it have to be based on the restriction that the traveller's overall utility does not change substantially. Within the context of VOT estimation, this assumption is generally warranted, but it must be recognised that in some planning contexts the assumption cannot be maintained and aspects of the theory, e.g. the constant marginal utility of money, may need to be abandoned.

---

4 'Truncated' means that only the relevant parts of the overall utility are considered.

The micro-economic theory thus gives a good basis for the specification and interpretation of models that can be used for time value estimation. It is important however to note that the theory given above applies to a single traveller or consumer only. The possibility of interpersonal variation is recognised in the SVT theory, but enters explicitly only when the SPT is introduced. All elements of the theory, explicitly including the marginal utility of money, may vary between individuals.

What is meant by the estimation of VOT is then the estimation of the ratio of the partial derivatives of the indirect utility function. A series of observations is required, in which consumers or travellers are observed making decisions between discrete alternatives. The fact that one alternative is preferred to another implies that the (indirect) utility of the chosen alternative is higher than those of all of the rejected alternatives. Given suitable assumptions about interpersonal (or *intra*personal) variation, estimates can be derived of the ratios of these coefficients: the appropriate methods of doing this are the main topic of the present paper.

In fact, travel data is quite suitable for estimating VOT, because the wide variety of origins and destinations observed in practical studies means that travellers are naturally faced with different levels of time, cost and other characteristics in the alternatives from which they are choosing. The variance this introduces facilitates the estimation process.

1.3 Structure of the Paper

The following Chapter describes the initial developments in VOT estimation, leading up to the adoption of the framework of utility maximisation with explicit errors that is currently used. Chapter 3 describes that framework with reference to the micro-economic theory summarised above. This is followed by overviews of a number of the current issues in VOT estimation, showing how each can be investigated within the standard framework. A concluding section summarises the main points of the paper.

2. INITIAL DEVELOPMENTS OF VOT ESTIMATION

The techniques described in this Chapter are those that were used in the early period of VOT estimation, approximately the sixties and early seventies, until the emergence of the current methodology before 1975. In assessing these early methods, it is important to remember that the costs of computation were very much higher in that period than they are now and that graphical and non-iterative methods were therefore used much more commonly for all kinds of analyses.

Early studies by SCPR (Hoinville, 1970) appear to foreshadow modern Stated Preference studies. However, I am not aware of any further work in that direction until the 1980's and the SCPR work certainly does not form part of the development of the modern methods.

2.1 The Beesley Graph

A review of literature on mode choice and value of time conducted by the Local Government OR Unit (Rogers *et al.*, 1970) identified work by Moses and Williamson

(1963) and by Beesley (1965) as among the earliest VOT estimations[5]. Of these, Beesley's work has clearly been more influential, in the UK at least, perhaps chiefly because of the highly intuitive graphical interpretation of the data.

Beesley worked with data of mode choice for workers in the then Ministry of Transport in central London. For a sample of these travellers the time and cost of the chosen mode were compared with the time and cost of the alternative[6]. From this data, the value of time was derived by a graphical technique.

The Beesley Graph (see Figure 1) interprets the choices made as being preferences either for a time saving or for a cost saving. For each traveller, the differences are calculated and one point for each observation plotted on a graph. A line is then drawn through the origin to maximise the number of points on the 'correct' side, i.e. below and to the left in the orientation of Figure 1. The value of time is then given by the angle of the separating line (specifically by the inverse tangent of the angle).

The Beesley Graph and its interpretation are clearly based on the concept of individual utility maximisation, i.e. rooted in micro-economics. The concept of 'trading' in cost and time is also clearly present. However, when applied in practice a number of problems arise:

- only some of the observations are relevant to the estimation of value of time, specifically those in the upper left and lower right quadrants of Figure 1, the 'traders'; the remainder of the observations, often the majority, are ignored as 'non-traders';

- several lines can often be found that correctly classify the same number of observations and there is no clear means by which a choice can be made among these;

- the traveller's choice is interpreted as based entirely on cost and (total) travel time; in practice it is known that several other characteristics of the journey are relevant and that, in mode choice, many of the most important characteristics are not even measurable;

- it is difficult to extend the analysis to derive forecasts for a future situation.

A more precise and detailed analysis was therefore required, allowing further dimensions to be introduced into the analysis and permitting a complete optimisation to be conducted. The first technique applied for this purpose was discriminant analysis.

2.2 Discriminant Analysis

Discriminant analysis is a multivariate procedure developed primarily in biological studies. It is applicable where there are two distinct but mixed populations, and the

---

5 I do not have access to the original papers and rely on the LGORU review, which covers much of the important work published in English in the 1960's.

6 Presumably the alternatives were public transport and car, the latter still being feasible (but available only to richer workers) in the early sixties.

analyst wishes to make a classification of individuals to the correct population on the basis of easily observable characteristics. For example, a population may be comprised of members of two species, which can be distinguished infallibly only by destructive testing, but for which there exist simply measured characteristics (e.g. leaf width) that allow an imperfect classification to be made. Given a sample for which the simple characteristics and the true species are known, discriminant analysis can be used to set up functions to make an optimal assignment of further individuals based only on the observable characteristics. A key aspect of discriminant analysis was that it is multivariate, i.e. it can be applied in several dimensions, rather than being restricted to two dimensions as in graphical procedures.

The technique seems to have been applied first in VOT measurement by Quarmby (1967)[7]. The procedure is illustrated graphically (for a two-dimensional case) in Figure 2. Given the two populations in the sample for which the population membership is known, discriminant analysis can be applied to determine an optimal separating line. The parameters of this line (slope and intercept) are then interpreted as the value of time and the average utility difference between the alternatives (e.g. the 'mode handicap', λ in Figure 2). A further, more extensive application was made by LGORU (Rogers *et al.*, 1970) in the first attempt at a study of value of time that would have national relevance in the UK. Following Quarmby, LGORU developed the analysis in several dimensions, using the multivariate capability of discriminant analysis to value simultaneously several different components of time, such as 'in-vehicle', walking and waiting time.

Discriminant analysis offers a number of advantages, which explained its popularity in that period.

- The technique runs quickly on a computer, giving a single optimum result.

- The extension to several dimensions is straightforward.

- Statistics concerning the parameter estimates (Student's t) and the overall discrimination (Mahalanobis D-squared) are generated conveniently from the process. For these statistics standard tables are available so that the overall significance of results can be looked up simply.

- Applying the same distributional assumptions that are used for deriving the statistics, it is possible to derive forecasting models; these turn out to be logit models.

However, there is a significant technical disadvantage. The statistics are based on the assumption of multivariate normality in the base data, and that the two populations are distributed with equal covariance. This assumption is far from valid in most transportation situations. This failure of the assumption does not necessarily invalidate the optimality of the discrimination function, but it certainly invalidates the statistics giving the significance of the discrimination and parameter estimates.

---

7 Once again I am relying on the LGORU review.

More seriously, the assumption of causality implicit in discriminant analysis is inappropriate to VOT estimation. Discriminant analysis is designed for the case where the observed characteristics (e.g. time and cost differences, in the transportation case) are **symptoms** of an underlying, unchanging species. VOT estimation requires to be based on an assumption of utility maximisation, that is that the choice will change if the time and cost differences change sufficiently. The basis of the technique is entirely wrong for this application.

The forecasting procedure derived from a discriminant analysis is undermined by the failure of the distributional assumptions in the same way that the significance statistics are undermined. Moreover, there is no philosophical basis for forecasting, because the theory appears to suggest that changes in time and cost will result from choice changes, rather than *vice versa*. It is interesting to note, however, that **if** the distributional assumptions for discriminant analysis were valid, then the discriminant function would be identical to that of a logit model derived by assuming utility maximisation in the way described in Section 3 below. However, that logit model can also be derived and applied validly when the distributional assumptions do not hold. Thus even if the assumptions are true, discriminant analysis offers no advantage compared with logit modelling other than computational speed.

It should be noted that it may in fact be the case that the causality implicit in discriminant analysis is valid. That is, the measured time and cost may actually depend on the choices made, rather than the other way round. For example, someone who was a habitual car traveller could afford to live in an area poorly served by public transport. However, the requirement for VOT estimation is to assume that choice is the result of trading, i.e. to find situations where that causality can reasonably be expected to apply.

For VOT estimation, it is therefore necessary to return to an explicit interpretation of traveller behaviour as the maximisation of utility.

2.3 Score Maximisation Procedures

Given the problems encountered in the causality assumptions implicit in discriminant analysis, analysts returned to a more detailed assessment of the meaning of an observation. In the Beesley graph, choices can be interpreted as giving a 'limiting time value', a phrase introduced by LGORU, which had become the leading VOT research agency in the UK in the early 1970's (Davies and Rogers, 1973).

Davies and Rogers insisted on the interpretation of an individual's choice as giving information about an upper or lower limit on that individual's value of time. This concept abandons the discriminant analysis approach of postulating two distinct populations and returns to the Beesley-graph approach based on utility maximisation. Depending on which of the four quadrants of the Beesley graph the individual falls in, the type of information given can be classified as shown in Table 1. The estimation of a value of time for a population is then a matter of determining that value for which the maximum number of individuals' limiting time values are consistent. Up to this point their approach is identical to that of Beesley, except that less emphasis is placed on the graphical approach and more on grid search procedures.

Table 1: Limiting Values of Time

Preference for 1 $\Rightarrow$ disutility 2 exceeds disutility 1 $\Rightarrow$ $\nu.t2 + c2 \geq \nu.t1 + c1$

which implies one of

	if....	**called...**
'traders'		
1. $\nu \geq (c1 - c2) / (t2 - t1)$	**$t2 > t1, c2 < c1$**	**'time saver'**
2. $\nu \leq (c2 - c1) / (t1 - t2)$	**$t2 < t1, c2 > c1$**	**'cost saver'**
'non traders'		
3. no information	**$t2 \geq t1, c2 \geq c1$**	**'dominant'**
4. a problem	**$t2 < t1, c2 < c1$**	**'irrational'**

Davies and Rogers then went on to apply these same concepts to multi-dimensional analysis, testing several components of time and even the dependence of time values on the size of time savings.[8] In these multi-dimensional analyses the concept of limiting time value is less clear, but the interpretation of the data as showing that one set of times and costs was preferred to another - i.e. that the utility of the chosen alternative exceeds the utility of the rejected alternative - was strictly maintained.

In each case the criterion used by Davies and Rogers to find the best interpretation of the data was to maximise the number of observations consistent with the hypothesis. This criterion of maximisation has been used in many other contexts, where it is known variously as 'percentage right', 'first preference recovery' and 'score maximisation'. It has been shown to be a poor criterion for comparing the overall success of models across different data sets, but in this context it is used as a criterion of optimality of a model on a single data set.

Under the name of score maximisation, Manski (1975) made a full investigation of the criterion, showing (for example) that the coefficient estimates derived by this criterion were **consistent**, that is that the estimates approach the true values as the amount of data becomes very large. The justification offered by Manski can be applied retrospectively to the Beesley-graph procedure, despite the graphical estimation method usually applied.

While the theoretical base in utility maximisation and the statistical verification of the method gave a better basis for VOT estimation than had been available previously, a number of difficulties are inherent in score maximisation.

---

[8] VOT appeared to increase with the magnitude of savings, but the work was based on mode choice, which reduces the relevance of the work to the modern debate on this issue.

- The programming is difficult and unreliable, partly because the optimum score value can be obtained for many different sets of parameter values in data sets of small or moderate size.
- Score maximisation gives a poorer fit to data than a more 'reasonable' assumptions with an explicit error assumption.
- Score maximisation gives no statistics or error measures for the estimated parameters.
- The method is very difficult to extend to more complicated choices.
- Score maximisation gives no explanation of 'wrong' observations, they are simply inconsistent with the model.

For these reasons, further improvements were sought in the estimation procedure.

2.4 Derivation of Present Estimation Procedure

In the early 1970's, parallel to the work on VOT estimation described above, developments were also taking place in the development of models of choice of travel mode and other choices relevant to transport planning. A historical review of this work, which mainly took place in the United States, is not possible here. I can do no more than list the main researchers: McFadden, Ben-Akiva, Manski and their associates. There were undoubtedly connections between the work of these people, but there was at that time much less trans-Atlantic communication than now and the work of Williams at Leeds and our work at LGORU proceeded independently of the Americans and indeed of each other.

As a result of this work, it was fairly widely known before 1975 that it was possible to construct models for binary and simple multiple choice of a number of forms that were consistent with a theory of individual utility maximisation, for example as outlined by Daly and Zachary (1975b). The difficult problem exercising researchers at that time was how the model could be extended to more realistic models of multiple choice without losing its basis in utility maximisation, a problem that was solved four times independently shortly afterwards. However, for VOT estimation the complexities of multiple choice were not immediately relevant, as almost all VOT estimations, even up to the present day, have been conducted on binary choices.

The existence of these models and their consistency with micro-economic theory disproved the assertion of (e.g.) Davies and Rogers (1973) that logit or probit models were "empirical classification procedures" and therefore not suitable for VOT estimation. The separation Davies and Rogers drew between choice modelling and VOT estimation disappeared and it became possible to employ choice models for VOT estimation, providing the models could be shown to be consistent with individual utility maximisation. The first application of this insight for VOT estimation of which I am aware was by Daly and Zachary (1975a).

The binary models that were found to be acceptable within the framework of utility maximisation represented choice on the basis of the comparison of the (indirect) utilities of the alternatives, into which a random element was introduced. For this reason the models are often called 'random utility' models. However, this naming is confusing, because the model does not require the utility itself to be random, only that the analyst can make only an approximation to the true utility and the difference between that approximation and the true utility can be treated as a random variable, i.e. no random component of behaviour is postulated. The randomness is thus in the sampling of particular individuals for analysis, rather than in their behaviour. An explicit distribution can be specified for the distribution of the approximation error and the model form can be derived from that distribution, as is described in the following section, to obtain models such as the logit or probit.

Given the greatly increased freedom given by the possibility of modelling, it became necessary to evaluate the position within the new framework of the score maximisation models, the only previous models that could still be considered valid. Comparing the explanation of the data given by a score maximisation method with that given by a method with an explicit error distribution, such as those in current use, is not easy because the two approaches do not apply comparable measures. One comparison, given by Daly and Zachary (1975a) is to convert the score maximisation method to a maximum likelihood method by postulating a suitable distribution of the unobserved utility component. They showed that (for one data set) this comparison showed a substantially worse explanation of the data (lower likelihood) than that given by a simple logit model. For this reason, along with the inherent difficulties of the score maximisation method listed in the previous section, score maximisation has not been used to any great extent for VOT estimation since the mid 1970's.

A further problem that was solved by the introduction of the models with an explicit error distribution was that of the traders and non-traders. It had been argued that 'non-traders' should be excluded from the data since they gave no information about time values. However, as argued by Daly (1978) the introduction of a random component into the model terminates this discussion: the non-traders cannot be identified and the analysis must be conducted on **all** the data. Travellers whose choices are much better than the alternatives will automatically contribute little in a well-formulated model.

3. UTILITY MAXIMISATION WITH EXPLICIT ERROR DISTRIBUTION

For the reasons explained in the previous section, VOT estimation in the last 20 years has been based almost exclusively on the interpretation of observations as being instances of individual utility maximisation, together with an explicit statement of the distribution of the non-measured 'error' term. This approach is based therefore on two key postulates:

the interpretation of a choice is that the chosen alternative has greater utility than the rejected alternative;

but

the analyst cannot observe <u>all</u> of the relevant determinants of the individual's choice.

These two basic points then determine the basic analytical features of the models that can be applied for VOT estimation.

Much of the information in the following sections can also be found in standard tests, such as Ben-Akiva and Lerman (1985). It is restated here from the point of view of VOT estimation, which is a little different from the approach needed for estimating forecasting models, the main interest of the standard texts. The basic theory is however identical.

3.1 Analytical Statement of the Model

The most important features of the analytical approach follow from the two postulates given above.

- All the relevant **measured** characteristics of alternative are included in the model. Omission of a characteristic that is relevant to the choice simply exposes the model to the risk of bias. Therefore as many of these characteristics as possible should be measured, although it is necessary to recognise that 100% coverage will **never** be possible.

- There is an explicit treatment of the inability of the analyst to measure all of the relevant characteristics for an individual (or to formulate the individual's (indirect) utility function correctly) and of inter-personal variation. This treatment takes the form of specifying the probability distribution of the difference between the true utility and the measured part of the utility.

- Because of this probability distribution of the unmeasured component, statistical procedures are required to define the optimality of the model, estimating the coefficients and their error.

The model is then based on the formulation for each alternative i of an indirect utility function U_i of the form

$$U_i = \beta_t \, . \, t_i + \beta_c \, . \, c_i + \ldots + \varepsilon_i \qquad (\beta < 0, \text{ usually})$$

in which

	t_i	is the time required when choosing alternative i;
	c_i	is the cost of alternative i;
	β	are coefficients to be estimated, usually negative because increasing time and cost usually reduce the utility of an alternative;
	...	indicate the other relevant measured characteristics of alternative i, probably also involving coefficients that require to be estimated;
and	ε_i	contains *all* the unmeasured difference between the measured components and the true utility.

If the only components of U that vary with the cost and time are c and t, then it is easy to see that the value of time, the ratio of the partial derivatives of U with respect to those variables, is just β_t / β_c. However, when other relevant characteristics vary with time or cost, the calculation will be more complicated[9].

Formulations as above are the basis of almost all modern VOT estimation procedures. However, the practicalities depend to a large extent on the assumptions that are made about the distribution of ε and on the types of data available. The main issues arising in each case are discussed in the two following sections.

3.2 Types of Data Used

VOT estimation is based on observations of individual behaviour. Given a sample of observations that can reasonably be described as representative of the total travelling population, the VOT estimates derived can reasonably be described as estimates of the average VOT of the total population, following the best available scientific methods.

The data that has been used can be classified in four main groups:

- **RP: revealed preference**: what travellers actually did: it may then **definitely** be concluded that the chosen alternative has a utility higher than those of all the rejected alternatives;

- **SP: stated preference** (generally using choice experiments[10]): what travellers state they would do in a hypothetical case: the preferred alternative is then **assumed** to have a higher utility than rejected alternatives;

- **TP: transfer price**: how much travellers say they would need to be compensated for using an inferior alternative: the transfer price is then **assumed to be correlated** with the true utility difference;

- **other preference data** (hedonic scales etc. and other forms of SP data): has not been much used in VOT estimation.

Because of the dominance of RP, SP and TP data in practical VOT estimations to date, only these three data types are considered further.

The objective of VOT estimation is primarily to estimate the coefficients of the indirect utility function, not to make forecasts of behaviour in changed contexts. For this reason, binary choices are preferred because they simplify both the data collection and the modelling contexts as far as is possible. The majority of VOT estimation has been performed on binary choices, although SP ranking data has been used occasionally.[11] In the remainder of the paper, binary data only will be considered.

---

9 This may well also apply when ε varies with time or cost.

10 It would be more specific to describe this type of data as 'stated choice', but the 'SP' label is very widely used and will be retained for this rerason.

11 That is, data in which the respondent reports the order of his or her preference over a set of real or imaginary alternatives presented in the experiment. The analysis procedures for data of this type can become complicated if full attention is given to intercorrelations.

RP data has the great advantage that the behaviour observed is real. The travellers have actually made the observed choices. For this reason RP data is often used to control other data types. However, in RP data survey difficulties mean it is often difficult (or even impossible) to observe types of behaviour that are of interest and the data may often be expensive per observation, with high correlations between the variables of interest, in particular of course between time and cost. For these reasons errors in VOT estimated from RP data are often unacceptably high and other data is needed to achieve adequate accuracy.

An issue arising in the analysis of RP data is how the times, costs and other relevant characteristics of the alternatives should be measured. One cheap method that has often been used in practice is to ask individual respondents what were the times and costs of their chosen and rejected alternatives; data of this type has sometimes been called 'perceived', although 'reported' is a more accurate description. The alternative, calculating the characteristics independently, i.e. what are often called 'engineering' estimates, is often more expensive, whether the characteristics are obtained from transportation network analysis or 'by hand'. However, the biases, such as self-justification, and lack of information, particularly about non-used alternatives, make the use of reported data dangerous.[12]

SP data is usually much cheaper than RP data to collect per observation, largely because of the practice of collecting multiple responses per individual. It has the further advantages that the analyst can design the correlation structure and ask the respondent to consider new situations that have not yet arisen, both of these within the bounds of what can be described credibly. The inherent disadvantage of SP data is that its relationship to real behaviour is in principle unknown, although by now there is substantial experience in the comparative properties of RP and SP data. For this reason SP data is often supported by RP data in a simultaneous estimation process. A further problem in SP practice is that the proper analysis of multiple responses per individual requires more sophisticated analysis procedures than those needed for RP data.

For RP data it is known, and for SP data it is assumed, that the observation that has been made implies that the indirect utility of the chosen alternative exceeds that of the rejected alternative:

RP, SP: Choice or preference is 1 $\Rightarrow$ $U_1 - U_2 \geq 0$

This is then the form in which the data is analysed.

TP data is analysed in the same theoretical utility-maximising framework used for RP and SP data. The advantage given by TP data is that it contains in principle more information per observation, because the respondent states not only which alternative is preferred, but also by how much. The central issue is whether respondents can understand the TP question, for which there are several different formulations which have proved variously successful in practical studies. The chief problem in practice,

---

[12] It is important not to confuse reported data with 'perceived' levels of service. If travellers do actually perceive their alternatives in terms of times and costs in specific amounts, they are certainly unable to report these without error and/or bias. Moreover, the ultimate interest is not in the value of perceived time but in the value of time as it can be measured in base and future contexts.

however, has been the difficulty of dealing with the additional information that the TP is always positive, because it represents the difference between the inferior and the chosen alternatives: if this information is not handled properly, the analysis can be biased. Further discussion of this issue is given below.

The analytical formulation for TP data is set up as

TP: TP from 1 to 2 $\Rightarrow$ $TP = \tau\,(U_1 - U_2) \geq 0$

in which τ represents a transformation of the utility difference which is in principle unknown at the start of the analysis.

Note that whether RP, SP or TP data is used, the analysis is based solely on the utility **difference**. This is as it should be, because utility has of its nature no well-defined zero point. Similarly, the scale of the utility is not defined (in TP data, the τ function may contain an arbitrary scale factor), reflecting the absence of a well-defined scale for utility.

3.3 Estimation Procedure

The essential step of the estimation procedure is to turn the formulation of the observations described in the previous sections into a statement of the probability of the observed behaviour. The probability statement can then be used in a statistical estimation procedure to derive the 'best' values of the coefficients of the model. The use of a statistical approach is an appropriate way of handling the uncertainties inherent in the observation of imperfectly understood phenomena such as travellers' choices.

In practice, the statistical procedure most commonly applied for VOT estimation is that of maximum likelihood. It is not possible here to explain fully this procedure or its advantages, which are substantial. The reader is referred to the statistical literature for this purpose (e.g. Cox and Hinkley, 1974). Among the advantages of the maximum likelihood method are that it yields statistics on the accuracy of estimation of the coefficients and gives the possibility of making tests to compare the 'fit' of competing models formulations.

The maximum likelihood criterion needs to operate on the **joint** processes of sampling individuals and their choice processes. However, in many cases, such as that of choice-independent sampling[13] it can be shown that it is sufficient to consider the **kernel** of the likelihood function, which for choice models as used for RP and SP analysis is

$$L = \Pi_o \, p_{co} \quad \text{or} \quad \log L = \Sigma_o \log p_{co}$$

where p_{co} is the probability of making the observed choice c for observation o. The logarithmic form of the function is usually more convenient. The maximum of the

[13] With so-called 'choice-based' samples it is often sufficient to adjust the **constants** of the models only, i.e. naïve analysis gives correct values for most of the β coefficients - see Ben-Akiva and Lerman (1985).

likelihood is of course obtained by the same parameter values as the maximum of the log likelihood.

It is therefore necessary to be able to calculate the choice probabilities p_c that the sampled individuals will make their observed choices.

For TP data the likelihood function has a different form, as is discussed below.

RP and SP Data

In RP and SP data, the probability of the traveller making the observed choice is the probability that the utility of the chosen alternative was greater than the utility of the rejected alternative. This means

$$U_1 - U_2 \geq 0$$

which implies, in terms of the approximations to the utility functions described above, that

$$\beta_t . (t_1 - t_2) + \beta_c . (c_1 - c_2) + ... + \varepsilon_1 - \varepsilon_2 \geq 0$$

or

$$(\varepsilon_2 - \varepsilon_1) \leq \beta_t . (t_1 - t_2) + \beta_c . (c_1 - c_2) + ...$$

It is therefore possible to state that the probability of choosing alternative 1 is

$$p_1 = \Pr \{ \text{choice} = 1 \} = F (\beta_t . \Delta t + \beta_c . \Delta c + ...) = F (V)$$

where Δt, Δc represent the differences in time and cost between the alternatives (1 - 2) F is the cumulative form of the probability distribution of $(\varepsilon_2 - \varepsilon_1)$ and V is the difference in the measured utility components between the alternatives.

If the form of F is known, then the values of β can be adjusted to obtain the best overall fit to the data in terms of the required optimality criterion (i.e. to find the maximum of the likelihood or log likelihood function). This process is the core of the estimation.

TP Data

The theoretical basis for analysing TP data is the same as that for RP and SP data. However the formulation of the model for analysis is of course different because of the form of the information available. The basic model is

$$TP = \tau (V) + (\varepsilon_2 - \varepsilon_1) \qquad \geq 0$$

where TP is the transfer price given by the individual respondent,

V, ε have the meanings described in the RP/SP discussion above and

τ is an unknown translation from measured utility difference to reported transfer price.

If simple assumptions are made about the form of τ (e.g. that it is linear) and about the distribution of (ε_2 - ε_1) (e.g. that it is normal), simple modelling methods can be used to estimate the β coefficients within V by the maximum likelihood criterion (e.g. ordinary least-squares regression[14]). More complicated methods can be used to handle more complex assumptions.

The problem that has arisen in the analysis of transfer price data is in the use of the information that the transfer price from the chosen alternative to an inferior alternative must be positive. For example, if the model is set up to compare the chosen with the rejected alternative, it cannot be assumed that the distribution of the error term is strictly normal, as this would attribute a positive probability that the rejected alternative had higher utility than the chosen. More seriously, there is inevitably a positive preference for each alternative among the part of the population that chooses it. Erroneous use of this information has led to biases in the results of the analyses and as a result TP data has been used less than it might have been. In general, it is preferable to set up TP data in the form of differences between alternative A and alternative B, applying a suitable sign to the transfer price information to indicate whether it relates to a transfer from A to B or from B to A.

Direct estimation

Whichever form of data is used, the measured component of the utility function can be reformulated as

$$V = \beta_c \,.\, \{ \nu \,.\, \Delta t + \Delta c + \ldots \}$$

in which the value of time ν (= β_t / β_c) appears directly as a coefficient in the model rather than being calculated subsequently from estimated coefficients.

This reformulation can be convenient in yielding the value of time (and the money value of other characteristics of the alternatives) directly. In particular, the error in the VOT estimates can often be estimated much more easily in this way than in a subsequent calculation.

The suggestion of direct estimation using choice data in this way was made by Bradley and Daly (1993), who showed that, if the base model was logit, this reformulation could be set up as a ‘tree’ logit.

14 For a regression with a normally-distributed error term, the maximum likelihood estimate is the same as the least-squares estimate.

3.4 Sources of Variation

The form of the distribution of $(\varepsilon_2 - \varepsilon_1)$ determines the form of the model that is to be estimated and the practicalities of the actual estimation process. Essentially, $(\varepsilon_2 - \varepsilon_1)$ represents the error in the analyst's understanding of the behaviour of an individual traveller. Additionally, it is necessary to take account in the modelling of sources of inter-personal variation. These two issues are considered in turn.

Distribution of $(\varepsilon_2 - \varepsilon_1)$

First, it is reasonable to assume that the mean value of $(\varepsilon_2 - \varepsilon_1)$ is zero. This is effectively the assumption that there are sufficient constants in the model. In simple cases it is also reasonable to assume that $(\varepsilon_2 - \varepsilon_1)$ has a symmetrical distribution, because it could be expected that the distribution of the individual ε variables was identical. However, in more complex models it is necessary to abandon this assumption, as will become apparent.

Second, the distribution of $(\varepsilon_2 - \varepsilon_1)$ can be considered to be induced by the distribution of the separate ε variables. Two particular and important cases will illustrate how this works.

1. If the separate ε's have normal distributions, then $(\varepsilon_2 - \varepsilon_1)$ will also have a normal distribution and the probability model with be of the probit form.

2. If the separate ε's have extreme-value distributions of a particular type[15] with equal variance, then $(\varepsilon_2 - \varepsilon_1)$ will have a logistic distribution and the probability model will be of the logit form.

Other assumptions for the underlying distributions could also be postulated, leading to other forms of distribution of $(\varepsilon_2 - \varepsilon_1)$, but these have not been used widely in practice, with the exception of the log-normal value of time distribution discussed below.

The underlying normal distribution is theoretically appropriate if the utilities of the alternatives are considered to be composed of sums of numerous components, since the normal distribution is the limiting distribution of a sum of a large number of components. Conversely, if the alternatives are considered to result from choice from a number of more detailed alternatives, e.g. choice over time of departure, route etc. for the car alternative, then the limiting-value distribution is more appropriate and the logit model theoretically justified. In reality, aspects of both of these arguments apply and the choice of appropriate distribution is better made on grounds of practicality. In most cases, this will lead to the use of the logit model, which is generally simpler to handle than the probit, but in some cases the probit turns out to be more tractable.

Third, the variance of $(\varepsilon_2 - \varepsilon_1)$ can be considered to be constant, giving simple models, or can be considered to vary. The variation might depend on the data source (e.g. RP and SP data, when analysed together, will often be found to have different error variance). Alternatively, the error variance might depend on aspects of the data, e.g. there may be more variance associated with long trips than with short trips, measured

[15] Also called the Weibull or Gumbel distribution.

on a constant scale. This type of heteroskedasticity (changing variance) is associated primarily with what is called ‘taste variation’.

Fourth, the errors for the separate observations may be considered to be independent or to have some correlation. The former, which is obviously much more simple in its modelling implications, is acceptable when the observations are taken from separate individuals, as is the case with most RP and TP data. However, when there are repeated observations from individuals, as is common in SP data, the modelling should take account of the correlations thus introduced.

The possibility of introducing more sophisticated variance assumptions is the basis for many of the more advanced VOT estimations.

Inter-personal variation

In addition to the error ε between the traveller’s true utility function and the analyst’s approximation of that function, it is necessary to take account in many cases of variation between individuals. There are a number of ways in which this can be done.

Most obviously and most simply, time valuations have always been considered to be different between business and leisure travellers, and frequently further differentiations are made in travel purpose, such as isolating commuter trips.[16] This issue has usually and appropriately been handled by separating travellers for different purposes and estimating their VOT separately. Obviously modelling a number of segments increases the data requirements for a study but this is inevitable because of the widely differing values that are found.

It is also possible to introduce into the utility functions variables that depend on the socio-economic description of the traveller. In particular the income of the traveller is often used to introduce variation in the VOT, an effect which can be achieved in various ways, some of which have more theoretical justification than others (see Jara-Díaz, 1996a). Other variables can also be introduced in this way to differentiate the population in a less thorough way than complete segmentation, requiring less data than that method.

These differentiations are based on observable characteristics. In recent research it has also proved possible to introduce interpersonal variation based on non-observed or ‘latent’ variables. The methods used for this are discussed along with other research issues in the following section.

4. CURRENT ISSUES IN VOT ESTIMATION

The model based on individual utility maximisation with an explicit error distribution described above is used for almost all current estimations of VOT, as is demonstrated by the other papers in this volume. The aim of the present Chapter is to show how the

[16] The issue of an employer’s valuation of time, when this cannot be deduced from the employee’s behaviour, is not covered in this paper. Other papers in the volume do deal with this issue and the valuation of time spent by goods in transit, which is clearly very relevant to the valuation of time for goods vehicles.

most important current issues are handled in that framework. These issues fall into three main groups:

- the treatment of data of different types;
- the specification of the utility function;
- the definition of the error term and the interpersonal variation.

These three groups of issues are considered in turn, followed by a discussion of the use of VOT information in forecasting.

4.1 Data Utilisation

The papers in this volume by Ortúzar and by Sheldon discuss many of the issues arising in the design and conduct of surveys for VOT estimation and here it is necessary to present only the aspects of data most closely related to estimation issues.

TP Data

We have already mentioned the problem that has restricted past use of TP data: the biases caused by incorrect treatment of the information that the sign of the TP is always known because the traveller's choice is always known.

A proper treatment of TP data is quite possible and when properly analysed it can be shown that the information content of TP data is, as expected, greater than that of choice data of RP or SP form (Gunn, 1984). The analysis needs to take account of the possibility of an 'inertia' effect in travellers' behaviour, but this effect is not always important, as Gunn shows on one data set.

Considering the advantages which can be obtained by the use of TP data, it appears that inadequate use as been made of this data type.

Design Issues

The issue of experimental design has generated a substantial literature and space does not allow a full discussion of the issues here. Research is continuing at Leeds and in other places (Watson *et al*., 1996).

The objective of experimental design is to increase the 'efficiency' of the estimation, that is, to minimise the estimation errors for a fixed amount of data or, alternatively to minimise the amount of data required to achieve a given level of accuracy.

With RP and TP data, the scope for improving design is limited. The most important variation is the choice of experimental context: location, behaviour studied, etc.. It has been suggested that some benefit can be obtained by preliminary interviews to select respondents for interview, but the gain from such a procedure will be small because a large part of the interview cost is in contacting the respondent, while an RP interview is typically very short.

With SP data, the scope for design improvements is considerable. One of the issues that is relevant here is the balancing of ‘extreme’ and ‘close’ decisions for the respondent. Further it is necessary to consider the appropriate correlation between the attributes of the alternatives: correlation is not necessarily unwelcome, particularly when ratios of coefficients are being estimated and not the coefficients themselves.

An over-riding concern, however, is to ensure that the alternatives presented to the individual are realistic.

SP Data

SP data has been used increasingly intensively in transport planning for 10-15 years now, proving particularly useful in estimating the ratios of model coefficients, which suggests its suitability for VOT estimation.

The advantage of SP data is primarily its cost-effectiveness. Experiments can be designed (as noted above) to maximise efficiency, while multiple responses can be obtained from each respondent, greatly reducing the cost per response. Further, the information assumed to be known to the respondent can be controlled by the interviewer, eliminating the need to use ‘engineering’ or ‘reported’ data which is necessary in the RP context. Finally, the behavioural context can be controlled, allowing the investigation of contexts and effects that are not observable in reality.

On the other hand, despite its popularity, not all of the problems of SP analysis have yet been solved. It has been known for some time that biases, such as inertia, may be present in the responses and that the error structure is significantly different from that of RP data. Following the pioneering work of Ben-Akiva and Morikawa (1990), convenient adjustment procedures for these features are in current use (Bradley and Daly, 1991). Less widely known are the problems that can arise with ‘adaptive’ SP (Bradley and Daly, 1993) or the difficulties that can be caused when the number of observations collected from a particular individual increases (Bradley and Daly, 1994). Nevertheless the ability of SP to collect repeated observations from an individual is one of its key advantages, reducing survey costs significantly.

A more fundamental problem is that the SP observations collected from an individual obviously cannot be described as independent as is required for simple analysis. While the intercorrelations that exist do not seem to affect the coefficient estimates themselves very much in practical situations, they certainly do affect the estimates of the errors in those estimates. Specifically, the use of naïve estimators substantially overstates the significance and accuracy of the results. A number of methods have been developed to deal with this issue. The most complete is to set up the likelihood function that is maximised to take full account of the intercorrelations; however, this requires specialised and complicated programming. Alternatives which require less (but still some) complicated programming are the adjusted matrix proposed by McFadden (1996), or the sequential modelling method of Ouwersloot and Rietveld (1996). A further reasonably simple approach is the ‘Jack-knife’ method proposed by Cirillo *et al*. (1996) which deals with the repeated measures problem and also takes account of other specification errors that may be present in the data.

The most fundamental issue with SP data, however, is the interpretation of the individual respondent's intention in answering the questions posed. It seems reasonable to suggest that the thinking in the few seconds taken to state a preference cannot incorporate a full optimisation of the utility over all possible allocations of time, money and travel choices. Therefore the responses must be taken as essentially short-term in nature and as giving an approximation to what might be observed in an RP survey, were it possible to conduct one for the same behaviour as the SP survey.

It may be concluded that SP data is an essential component of the analyst's armoury for VOT estimation. However, the analysis must be conducted in the knowledge of the biases and difficulties arising in SP analysis and the results must be seen in an essentially short-term context and only as approximations to the longer-term, more fully optimised, behaviour that can only be observed in RP data.

4.2 Form of the Utility Function

A further set of current issues concern the form of the utility function: the variables that should be incorporated and the form in which they should appear. While it is possible to draw some information from theory, many of the decisions concerning these variables need to be based on estimation results. The estimations are conducted by extending the (indirect) utility function difference as follows

$$V \quad = \quad \beta_c \,.\, (\nu \,.\, \Delta t \,+\, \Delta c \,+\, \nu_1 \,.\, \Delta x_1 \,+\, \nu_2 \,.\, \Delta x_2 \,+\, ..\,)$$

where the Δx's represent the differences between the alternatives of the additional variables and ν_1, ν_2 are the marginal values of these additional variables (measured directly in money terms because the whole function is multiplied by β_c). The function is shown with two such new variables but of course any number can be added that the information content of the data will support.

The tests that are made concerning the proper formulation of the function can be based on the underlying statistical theory of the estimation and in particular of the maximum likelihood estimation:

- a test of the inclusion of a single variable can be most conveniently made using the 't' test, part of the output of standard estimation software;

- a test of two models that are related to each other in a more complicated way, but which remain 'nested' (one model is a generalisation of the other) can be compared using the χ^2 test based on the difference in likelihood between the two models.

Standard tables are available for both 't' and χ^2. For more complicated situations, when models are not strictly nested, the likelihood value can be used to compare models estimated on the same data. Comparisons of models based on different data is more difficult, but some guidance can be obtained from the ρ^2 statistic, which is derived from the likelihood and which gives an overall assessment of the explanation of the data given by the model[17].

Variables to be incorporated

A wide range of variables can be considered for incorporation in the model in the way described above. These can be classified as follows.

- '**Comfort**'-type variables apply to different travel modes to explain the fact that, for example, car travel is generally considered less onerous than public transport travel. They can also apply to components of the journey by a single mode, such as congested or 'free-flow' driving, or walking, waiting and 'in-vehicle' time on a public transport journey.

- '**Person type**' variables can be applied in combination with time variables to persons of different types (e.g. by sex, age or employment status) or in different regions, or, most importantly in VOT estimation, with different incomes. Variables of this type can also be applied to distinguish different travel purposes, although this distinction is more often handled by separating the purposes altogether.

- '**Dummy**' variables based on person type or trip characteristics can be applied as 0-1 indicators to one of the alternatives to indicate a general preference for that alternative by persons of that type. These variables do not directly influence the VOT estimates, but by improving the quality of the model they can contribute to a general improvement to the estimation.

Variables of the comfort type can be handled in two ways. They can be set up by separating the time into distinct components:

$$V = \beta_c \,.\, (\Delta c + \nu_1 \,.\, \Delta t_1 + \nu_2 \,.\, \Delta t_2 + ..)$$

where t_1 would be the time spent under condition 1 (e.g. in a car) and t_2 the time spent in condition 2 (e.g. in a bus). The ν values estimated are then the absolute monetary values of time under those circumstances.

Alternatively, the function can be set up to estimate the **difference** in time value:

$$V = \beta_c \,.\, (\nu \,.\, \Delta t + \Delta c + \nu_2 \,.\, \Delta t_2 + ..)$$

in which t is the total time spent in one alternative and t_2 is time spent under particular circumstances (e.g. congested driving). Since the t_2 time is also included in t, ν_2 represents the **difference** in valuation from the standard time of condition 2. This latter formulation is often more convenient for testing how many different time components need to be valued separately, since the simple t test can be used to test the hypothesis that ν_2 is different from zero. However, the two formulations are entirely

---

17 The more familiar R^2 statistic used in linear regression is in fact a particular case of a ρ^2 statistic (if the residual error in the distribution is normally distributed); the two statistics can be interpreted in the same general way.

equivalent in terms of the explanation of behaviour they give and would have identical likelihood values.

Person-type variables need to be combined with time variables to influence VOT estimates directly. These variables can also be set up in two ways: either

$$V = \beta_c \,.\, (\Delta c + \nu_1 \,.\, \Delta t \,.\, \delta_1 + \nu_2 \,.\, \Delta t \,.\, \delta_2 \; ..)$$

or

$$V = \beta_c \,.\, (\nu \,.\, \Delta t + \Delta c + \nu_2 \,.\, \Delta t \,.\, \delta_2 + \; ..)$$

in which δ_i indicates the membership of a particular population group (1 for a member of group i, 0 for a non-member). The first formulation estimates a value ν_1 which applies to group 1 and ν_2 which applies to group 2. The second formulation estimates a value ν which applies as a general standard value and a further value ν_2 which represents the difference from the standard valuation by group 2. As with the comfort variables, there is no essential difference between the formulations and either can be used as appropriate to the situation. More than two proups can easily be tested.

Small time savings

A very important issue which is approached by varying the utility function is that of small time savings. The question is whether the marginal value of time savings depends on the magnitude of the time savings.

As in the case of the other variables that can be added to the utility function, small time savings can be introduced in an absolute or difference form, the latter being more suitable for testing whether there is a significant difference between small and other time savings. The absolute form would be set up as

$$V = \beta_c \,.\, (\Delta c + \nu_1 \,.\, \Delta t_1 + \nu_2 \,.\, \Delta t_2 + \; ..)$$

where $\Delta t_1 = \min (\Delta t, t_{lim})$

$\Delta t_2 = \dim (\Delta t, t_{lim}) = \max (0, \Delta t - t_{lim})$

and t_{lim} is the time up to which the difference is considered 'small'.

In this formulation, ν_1 estimates the value of time savings up to t_{lim}, e.g. 5 minutes, while ν_2 estimates the value of time savings over that limit.

While the formulation above seems straightforward enough, a few words of caution about the interpretation of the results are in order. First, similar tests could in principle be made for small cost savings; the theoretical argument concerning the constant marginal utility of money (made by Jara-Díaz, 1996b) applies equally to time. Second, it is difficult to collect RP data that relates to small time savings, so that estimations are nearly always carried out on SP data; the point made above concerning SP data, that long-term optimisation is obviously not possible, reduces the quality of the approximation to RP data in this case, since the traveller will be more quickly able to imagine an alternative use for a small quantity of money than for a small quantity of time. Finally, issues of heteroskedasticity (see next section) apply in the estimation of small time valuation, although it is not possible to be clear whether these have been of practical importance in the studies that have been made.

The conclusion of these points is that the estimation of the value of small time savings, and in particular the long-term value of those savings, is more difficult with current procedures than is the estimation of more substantial amounts of time.

4.3 Error Distributions

The main area of current work on the methodology of VOT estimation, as opposed to the derivation of practical results, is in the description of the distribution of the ε variables. To recap, the error is the difference between the analyst's approximation to the (indirect) utility function and the true value of that function for a specific individual. In practice, only the **difference** in the errors between the two alternatives is of relevance. It is reasonable to assume that the mean value of this distribution is zero. The remaining issues are then the form of the distribution and its variance. Of these, the variance is of primary importance.

When the error is not related to any of the measured variables, and all of the data comes from the same source, it is reasonable to assume that the variance is constant. This assumption leads to the simplest forms of model, such as those discussed in Section 3 above.

When data from different sources is analysed simultaneously, in particular when estimation is based on RP and SP data, it is necessary to take account of the different variances in the data sets; a scaling procedure is the standard method. For probit models the procedure of Ben-Akiva and Morikawa (1990) can be followed, for logit models this can be done by the use of a tree logit model as given in Bradley and Daly (1991).

The most complicated situations arise when it is required to relate the error variance to the measured values of the explanatory variables. For example, if the approximate utility difference is given by

$$V = \beta_t \,.\, \Delta t + \beta_c \,.\, \Delta c + \ldots$$

and it is postulated that the β values vary between the individuals, then the error variance is given by

$$\mathrm{var}(\varepsilon) = \underline{\Delta x}^T \,.\, \underline{\underline{B}} \,.\, \underline{\Delta x}$$

where $\underline{\Delta x}$ represents the difference in the t, c and other measured variables in the equation and $\underline{\underline{B}}$ is the covariance matrix of the $\underline{\beta}$ coefficients. Because this variance depends on $\underline{x}$, heteroskedasticity is introduced into the model and the estimation becomes complicated. While it was shown long ago that this heteroskedasticity could be important (Daly and Zachary, 1975a), even now there are few studies that take it into account. When the distribution of $\underline{\beta}$ is normal the form of the model for analysis is probit; other distributional forms will lead to more complicated analysis requirements which were not possible in early studies.

More recently, it has also become possible to introduce realistically skewed distributions, such as the log-normal VOT distribution presented by Ben-Akiva and Gopinath (1996). The advantage of these more sophisticated forms can be tested by the usual statistical tests.

A further source of latent interpersonal variation can also be introduced in the segmentation of the travellers. The 'latent class' model attributes a set of coefficients to each of a number of classes of travellers, but the class remains latent so that it is not known *a priori* to which class each traveller belongs. The models then proceeds in two stages: first class membership is predicted, then choice conditional on class membership, using for the class membership the set of coefficients appropriate to that class, in the following way:

$$\Pr \{ \text{choice} = 1 \} = \Sigma_k \Pr \{ \text{class} = k \} . \Pr \{ \text{choice} = 1 \mid \text{class} = k \}.$$

The probability of choice is the sum of the probabilities of membership of each class **times** the probabilities of making the choice **given** that the class is specified. Ben-Akiva and Gopinath (1996) show that this generalisation can give quite substantial improvements in the quality of the model. While the conditional models of choice are typically of a simple form, the class membership models can take more complicated forms - see the referenced paper for further details.

While all of these methods can be shown to improve understanding of the choice mechanisms, they also all introduce complexities of estimation which require specialised software and an advanced understanding of the estimation process. The value of the improved estimates has therefore to be set against the increased costs that are incurred in obtaining them.

4.4 Forecasting

The primary intention of the VOT estimations described in this paper has simply been to obtain model parameters for use in the appraisal of transport projects, such as road construction and public transport improvements. However, VOT estimates have also traditionally played an important role in the construction of forecasting models, in particular in the construction of mode choice models,

The basic approach that has been taken is to set up a model, usually of the logit form, with the utility functions defined by

$$V_m = \lambda_m + \beta (c_m + \nu_1 . t_{m1} + \nu_2 . t_{m2} + ..)$$

giving the utility function V_m for mode m as a function of an alternative-specific constant λ_m, a general scale factor β (< 0) and a 'generalised cost' function incorporating the cost c_m of mode m and the various component times for that mode t_{m1} etc., each multiplied by the appropriate VOT ν. If the ν values are taken from external sources, the requirement to collect local data for each study is considerably reduced.

The use of the generalised cost function in this way has a long history, in particular in the UK, where the key reference, introducing the concept as a practical planning tool, is MAU Note 179 (McIntosh and Quarmby, 1970). The use of the generalised cost function and limited local calibration became standard practice to deal with circumstances

- when local data was not available or was insufficient to estimate a local model;
- when the results of a local model were poor, perhaps because of excessive intercorrelations;
- to ensure inter-regional consistency of forecasting and planning decisions; or
- simply to check the local model.

Essentially, this process is one of 'transfer' of part of the model, the ν values, from one time and place to another time and place. Later, a better theoretical explanation of transfer and a practical 'scaling' methodology were developed (Gunn *et al*., 1985). It is interesting to note that the exploitation of the generalised cost function described above is simply a special case of the scaling method.

For practical studies, the process of transferring generalised cost functions based on standard VOT values has advantages, both in terms of the cost of local studies and in terms of the reliability of the VOT estimates themselves. However, local estimates of both λ and β do have to be obtained. The λ parameter typically presents few problems, since its function is to correct the overall mode split and local data on this is usually available. However, β, which represents the elasticity, is often difficult to calibrate locally. It is important to realise that SP (or TP) studies are of little help in estimating β: it is exactly the scale of the model, represented by β, that requires RP data to be estimated reliably.

In comparing the modelling needed for VOT estimation and for forecasting, it is important to be clear that the theoretical framework used in each case is the same: individual utility maximisation with an explicit error term. However, there are important practical differences between the work that is needed in the two cases, as follows.

- In VOT estimation, the objective is to simplify the analysis as much as possible to focus on the estimation of a small number of parameters. In forecasting, however, it is necessary to understand and represent the totality of the market and to estimate as many parameters as necessary to represent that market.
- For simplicity VOT estimations will usually focus on binary choice, while for comprehensiveness in forecasting multiple alternatives will generally be needed.
- Similarly, VOT estimation can focus on small groups of the population who have choice situations of particular interest, while forecasting needs to cover the entire range of choice situations.

- In VOT estimation, it is the 'trade-offs', the **relative** values of the coefficients, that are important. For forecasting it is the elasticities, defined by the **absolute** values of the coefficients, that are required.

The main impact of these differences is in the design of data that is appropriate for estimation in the two contexts. Further, the estimation methodology will generally be different for technical reasons, because the binary choice models used for VOT estimation can take account of complicated forms of interpersonal variation that would impose excessive burdens in the multiple-choice models required for forecasting.

However, the fact that the theoretical framework remains consistent means that information can be passed between the two contexts. In particular, VOT estimates can be imported into or used to check forecasting models and forecasting models can be used to yield VOT estimates, although these will not usually be as good as those obtained from a specialised study of VOT.

5. CONCLUSIONS

It is possible to define 'value of time' consistently with commonsense and economic theory. The theoretical definition establishes that the vague concept of the 'utility' function used in many analyses can be identified with the indirect utility function of microeconomics, from which values of time can be extracted to which a proper economic meaning can be attached.

The historical development of methodology led, after comparatively minor digressions, to utility maximisation with explicit error specification. The original interpretation of data formulated by Beesley is still used in principle, although of course the methods have been refined and extended enormously. The use of discriminant analysis has been abandoned.

The current estimation procedure is inherently statistical, which is an appropriate technology for handling incomplete observation. The analyst's approximation of utility is represented as differing from the individual's true (indirect) utility by an unknown amount, which is treated as a random variable in the analysis procedure.

The various data types - Revealed Preference, Stated Preference and Transfer Price - on which VOT estimations are based can be used for estimation within the utility framework, whether separately or together. The estimation itself is generally based on the maximum likelihood principle. Assumptions made about the distribution of the error term determine the form of model that is appropriate.

Almost all current VOT issues can be investigated within this framework; three areas are of particular interest.

- The combination of data of different types is employed to improve estimation accuracy. In particular interest focuses on the use of SP data, which is the most cost-effective estimation basis but which is still subject to a number of uncertainties in its interpretation.

- A wide range of variables other than cost and total time can be incorporated into the model to improve the explanation of behaviour and the estimates of time values. In particular the importance of ‘small time savings’ can be studied in this way, although some caution is needed in interpreting results based on SP data.

- Current methodological studies focus on improving the treatment of interpersonal variation, which can give substantial improvement in the explanation of behaviour but which requires more sophisticated models and more complex estimation procedures.

The estimated values can also be applied in forecasting models when local data is inadequate or not available. This application of values of time is essentially an application of a standard model transfer procedure and is justified by the literature on model transfers.

References

M. E. Beesley, ‘The value of time spent in travelling: some new evidence’, *Economica*, **32**, pp 174-185, 1965.

M. Ben-Akiva and D. A. Gopinath, ‘Estimation of randomly distributed value of time’, *this volume*, 1996.

M. Ben-Akiva and S. R. Lerman, *Discrete Choice Analysis: Theory and Application to Travel Demand*, MIT Press, Cambridge Mass. 1985.

M. Ben-Akiva and T. Morikawa, ‘Estimation of travel demand models from multiple data sources’, *11th. International Symposium on Transportation and Traffic Theory*, Yokohama, 1990.

M. A. Bradley and A. J. Daly, ‘Estimation of Logit Choice Models using Mixed Stated Preference and Revealed Preference Information’, presented to *6th. International Conference on Travel Behaviour*, Québec, 1991.

M. A. Bradley and A. J. Daly, ‘New analysis issues in SP research’, presented to PTRC Summer Annual Meeting, 1993.

M. A. Bradley and A. J. Daly, ‘Use of the Logit Scaling Approach to Test for Rank-Order and Fatigue Effects in Stated Preference Data’, *Transportation* **21**, pp 167-184, 1994.

C. Cirillo, A. J. Daly and K. R. Lindveld, ‘Eliminating Bias due to the Repeated Measurements Problem in SP Data’, presented to PTRC European Transport Forum, 1996.

D. R. Cox and D. V. Hinkley, *Theoretical Statistics*, Chapman and Hall, London 1974.

A. J. Daly and S. Zachary, *Commuters' Values of Time*, Local Government OR Unit Report T55, January 1975a.

A. J. Daly and S. Zachary, 'Calibration of Modal Split Models', presented to PTRC Summer Annual Meeting, 1975b.

A. J. Daly, 'Issues in the estimation of journey attribute values', in D. A. Hensher and Q. Dalvi (eds.) *Determinants of travel Choice*, Saxon House, Farnborough 1978.

A. L. Davies and K. G. Rogers, *Modal Choice and the Value of Time*, Local Government OR Unit Report C143, March 1973.

H. F. Gunn, 'An analysis of Transfer Price data', presented to PTRC Summer Annual Meeting, 1984.

H. F. Gunn, M. Ben-Akiva and M. A. Bradley, 'Tests of the scaling approach to transferring travel demand models', TRB, 1985.

G. Hoinville, 'Evaluating Community Preferences', SCPR, 1970.

S. R. Jara-Díaz, 'Time and income in travel demand', presented to Conference on Theoretical Foundations of Travel Choice Modelling, Stockholm, 1996a.

S. R. Jara-Díaz, 'Income, Leisure and Value of Time from Discrete Choice Models', *this volume*, 1996b.

C. F. Manski, 'Maximum score estimation of the stochastic utility model of choice', *Journal of Econometrics*, **4**, 1975.

D. L. McFadden, 'On the analysis of "intercept and follow" surveys', presented to Conference on Theoretical Foundations of Travel Choice Modelling, Stockholm, 1996.

P. T. McIntosh and D. A. Quarmby, 'Generalised Cost and the Estimation of Movement Costs and Benefits in Transport Planning', MAU Note 179, Department of the Environment, December 1970.

L. N. Moses and H. F. Williamson, 'Value of time, choice of mode and the subsidy issue in urban transportation', *Journal of Political Economy*, **71**, pp. 247-264, 1963.

H. Ouwersloot and P. Rietveld, 'Stated choice experiments with repeated observations', *Journal of Transport Economics and Policy*, pp. 203-212, May 1996.

D. A. Quarmby, 'Choice of travel mode for the journey to work', *Journal of Transport Economics and Policy*, **1 (3)**, pp 273-314, 1967.

K. G. Rogers, G. M. Townsend and A. E. Metcalf, *Planning for the Work Journey*, Local Government O. R. Unit Report C67, April 1970.

S. M. Watson, J. P. Toner, T. Fowkes and M. Wardman, ‘Efficiency properties of orthogonal stated preference designs’, presented to PTRC European Transport Forum, 1996.

Figure 1: Beesley-Graph
(Time and Cost Differences: Chosen minus Unchosen)

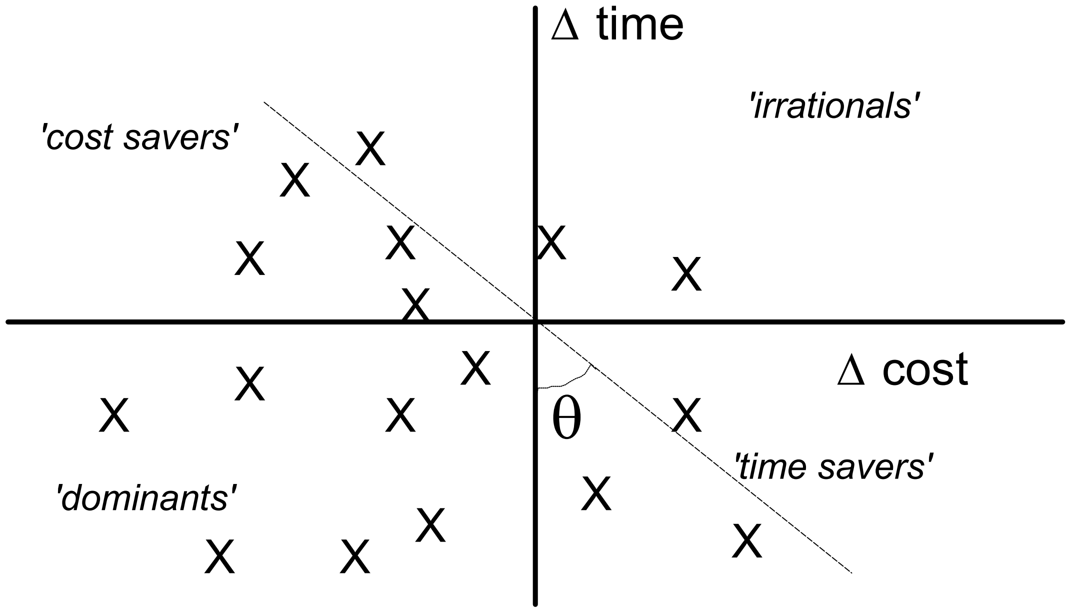

Figure 2: Discriminant Analysis
(Time and Cost Differences: Bus minus Car)